The Calculation of Scoring Resistance in Gear Drives

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Prediction of the oil film resistance in highly loaded gear drives: In the design of gear drives for high power transmission at speeds, it is necessary to make assessments regarding the so-called scoring limit[3].

Introduction

By “scoring” it is understood as a form of surface damage on the tooth flank, occurring when the oil film between the tooth flanks breaks down, and metal particles are torn out of one flank and welded onto the mating flank. This damage is particularly discernible in the form of streak marks on the flanks running in the direction of the tooth tips.

A scoring criterion involving a minimum of data concerning oil properties and surface quality is the analytically determined, so-called flash temperature expounded by Prof. Blok[1]: This states that the cause of scoring is the local temperature increase arising due to friction at point of tooth contact, when the temperature surpasses a value dependent on the oil viscosity and the type of oil (light or heavy E.P. oil). The frictional conditions prevailing between tooth flanks cannot be exactly imitated with rollers in experimental rigs, this is because the surface roughness of tooth flanks characteristic of their method of manufacture is comparatively large in proportion to the calculated oil film thickness, and so permits only partial hydrodynamic lubrication in the presence of significant leading[9]. This also explains why under such conditions, scoring is less influenced by the oil film thickness[2], than by a coefficient of friction dependent on the surface pressure and sliding velocity. The flash temperature criterion is found on equal considerations of tooth friction, and serves as a tentative prediction of the scoring risk. The following formulae afford an easily manipulated method of assessment in this direction.

Notation

Symbol	Unit	Term
a	mm	Centre distance
b	mm	Face width
c_n		Viscosity factor
c_R		Roughness factor
d_{b1}	mm	Pinion base circle diameter
d_{b2}	mm	Wheel base circle diameter
d_{a1}	mm	Pinion outside diameter
d_{a2}	mm	Wheel outside diameter
f		Instantaneous coefficient of friction
$f_1, f_{2,} f_{10,} f_{20}$		Auxiliary functions
k_1		Tooth contact parameter for pinion to tooth tip
k_2		Tooth contact parameter for wheel tooth tip
l	mm	Total length of lines of contact
n	min^{-1}	Rotational speed
p		Index
$Pmax$	kg/mm^2	Hertzian pressure
q		Index
u		Gear ratio $z_{2/}z_1$
v_r	m/s	Rolling velocity
v_r	m/s	Sliding velocity
v_w	m/s	Pitch line velocity

Notation

Symbol	Unit	Term
w_{tw}	kg/mm	Specific loading tangential to the pitch circle
z_1		No. of teeth in pinion
z_2		No. of teeth in wheel
a_{a1}		Pressure angle at pinion tooth tip
a_{a2}		Pressure angle at wheel tooth tip
a_{tw}		Working pressure angle (transverse)
β		Helix angle
ɣ		Specific sliding
ε_{a1}		Pinion addendum component of transverse contact ratio
ε_{a2}		Wheel addendum component of transverse contact ratio
e_a		Contact ratio
A_1,A_2		Auxiliary functions
C_{BP}		Characteristic for assessment of scoring risk
E	O_E	Lubricating oil viscosity in degrees Engler at 50˚C
F_n	kg	Load normal to the tooth flank
P_{lt}	kg/cm	Specific loading at tooth tip normal to flank
R	cm	Semi-width of Hertzian contact area
R_a	µ in.	Surface roughness
R_{am}	µ in.	Mean surface roughness
T_{BP}	˚C	Permissible flash temperature
T_{B1}	˚C	Flash temperature at pinion tooth tip
T_{B2}	˚C	Flash temperature at wheel tooth tip
T_B lim	˚C	Flash temperature at scoring limit
$\theta_{\max t}$	˚C	Flash temperature

The general relationship between scoring and flash temperature

Under the influence of surface load and speed during flank contact, friction causes a temperature rise at the load point according to the load, the speed, and the heat dissipation conditions. The original formula derived by Prof. Block for this so-called "flash temperature"[1] is as follows:

$$\theta_{\max t} = 0.83 \frac{\mathrm{f} \cdot P_{1t} \cdot (v_1 - v_2)}{(b_1 \cdot \sqrt{v_1} + b_2 \sqrt{v_2}) \cdot \sqrt{R}} \qquad (1)$$

$\theta_{\max t}$	Flash temperature in ˚C
f	Instantaneous coefficient of friction
P_{1t}	Specific loading in kg/cm
v_1, v_2	Flank speeds perpendicular to the line of action in cm/s
b_1, b_2	Heat dissipation characteristics
R	Semi-width of Hertzian contact band in cm

Since it is not possible to express the surface finish exactly, and hence the true nature of the surface contact cannot be completely specified, the calculated temperature rise does not correspond with the actually occurring peak temperature, but is substantially lower. The calculated value nevertheless serves its purpose satisfactorily, in that it has been revealed as a useful pointer for the scoring risk. After passing through the mesh, the surface temperature peak subsides again due to heat dissipation into the body of the tooth. The peak temperature is directly influenced by the tooth contact geometry, the load, the speed, the surface roughness, and the oil viscosity, but not the cooling of the gear drives by the lubrication oil. The latter might influence the basic temperature of the tooth, but nothing else. According to Prof. Blok, the governing temperature peak is obtained by adding the flash temperature to the mean temperature of the tooth body. In the following analysis of the resistance to scoring, consideration is limited to the flash temperature on account of its prime importance. A restriction of the forecast to a given set of manufacturing conditions is warranted, since gear tooth precision, assembly accuracy and surface finish can have considerable influence on scoring risk.

A scoring criterion to suit high capacity gear design

On the strength of experience gained in high capacity gear construction; more particularly of observations made in alignment of gear shafts, i.e. modification of load distribution across the face width (bearing patterns) and subsequent start-up, the following relationships between the scoring limit and load and speed have been determined for one and the same gear drive:

$T_{B\ lim}$ is proportional to Load x $\sqrt[4]{Speed}$

Where:

$T_{B\ lim}$ = Flash temperature at scoring limit

The geometric conditions, the surface roughness and the oil properties were thus excluded from the relationship. In the form of an equation:

$$T_{B\ lim} = w_{tw}^{n} \ . \ n^{q}$$

where

P= 1 and q = 0.25

The test results of Hughes and Waight[4] lead to values of:

p= 1 and q = 0.40

A perusal of the experimental report discloses, however, that this relationship did not hold for all speed levels. At higher speeds especially, presumably due to improved load carrying by hydrodynamic lubrication, the influence of the speed on the scoring load was lessened. Since high capacity gears usually operate at higher speeds than those cited in the stated tests, the load-speed relationship determined for them empirically seems reasonable.

If, with respect to the original formula for the flash temperature (which obeys the load-speed relationship

P = 0.75 and q = 0.50

with constant coefficient of friction) it is desired to derive a coefficient of friction corresponding to the empirical relationship p = 1 and q = 0.25, the subtraction of the indices yields:

f is proportional to $(\text{load})^{0.25}$ x $(\text{speed})^{-0.25}$

that is, to $\sqrt[4]{\frac{load}{speed}}$

From the investigations by Niemann, Ohlendorf[5] it can be concluded with reference to the diagram, Fig. 1, that this relationship for the coefficient of friction does in fact apply for higher loads. At lower load levels, the predominance of hydrodynamic loading leads to a slightly different relationship.

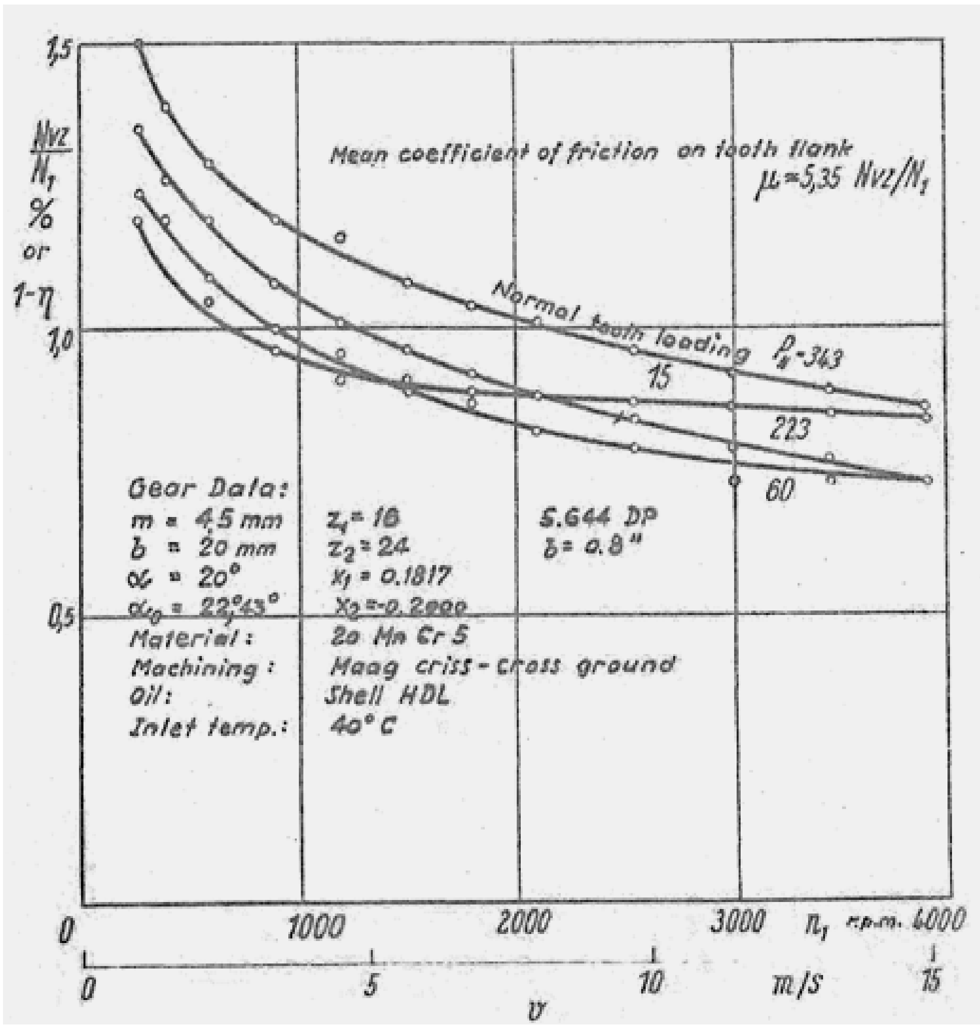


Figure 1: Losses in gears (from investigations by Niemann, Ohlendorf) in relationship to normal tooth loading and pitch line velocity

To allow for the variability of the coefficient of friction f in the formulas[1], in accordance with the load-speed relationship, the following algebraic expression can be substituted

$$f = \text{is proportional } \frac{\sqrt{f_{max}}}{\sqrt[4]{v_s}}$$

In the following calculations it is assumed that the highest flash temperature occurs at the tooth tip. Where profile corrections are applied it is necessary in each case to investigate whether the tooth tip point or the point of single tooth contact is the more critical[6]. For simplification the scoring criterion is referred to the flash temperature at the tooth tip of the pinion or wheel with non-corrected profiles. The specific loading normal to the tooth flank p_{lt} occurring at these points taken to be:

$$p_{lt} = \frac{w_{tw}}{\cos a_{tw}} * \frac{1}{\varepsilon_{\propto}}$$

For spur gears, this assumption coincides with the customary one for tooth bending calculations. For helical gears with an overlap ratio equal to or exceeding unity, the specific loading normal to the tooth flank can be derived from the tangential load and the total length of the lines of contact as follows:

Total length of lines of contact

$$l \approx \frac{b\ .\ \varepsilon_a}{\cos \beta_b}$$

Load normal to tooth flank

$$f_n = \frac{w_{tw}\ .\ b}{\cos a_{\propto w} \cdot \cos \beta_b}$$

Average specific loading along the lines of contact = specific loading & tooth tip normal to flank:

$$P_{lt} = \frac{F_n}{1} = \frac{w_{tw} \cdot b \cdot \cos \beta_b}{\cos a_{tw} \cdot \cos \beta_b \cdot b \cdot \varepsilon_{\propto}} = \frac{w_{tw}}{\cos a_{tw} \cdot \varepsilon_a}$$

With the assumption already made for the variable coefficient of friction, and with the specific loading derived above, we obtain values for the flash temperature at the tooth tips of pinion and wheel, in accordance with formula (1).

Flash temperatures obtained on the strength of the above stated assumptions would yield different scoring risk levels for pinion and wheel according to the gear ratio. Scoring would have to take place at a lower flash temperature on the pinion tooth tip than on the wheel tooth tip. Observations made in the behavior of high capacity gears, however, do not accord with this; the flash temperatures for the pinion and wheel tooth tips are equally responsible for the occurrence of scoring. The increased risk of scoring for the pinion tooth tip, as compared to the above theory, is associated with the more frequent meshing of the pinion teeth relative to those of the wheel, due to which the pinion acquires a somewhat higher temperature than the wheel. The difference in thermal expansion thereby caused results in a larger base pitch in the pinion compared to that in the wheel, whereas the pitches are equal in the cold condition. Hence load distribution is affected to the detriment of the pinion. The lubrication of the pinion addendum flank is also at a disadvantage due to the more frequent scraping of the adherent oil during the rolling/sliding action. Since lubricant reaches the critical scoring point chiefly via the addendum flank, it must be expected that the pinion tip point receives less oil than that of the wheel.

In order to apply the flash temperature method of assessment of scoring risk to pinion and wheel with one and the same critical scoring temperature, the flash temperature calculated in accordance with the above mentioned assumptions must be multiplied by a factor when applied to the pinion tooth tip. Experience with high capacity gear drives suggests $\sqrt[4]{u}$ to be a suitable, simple expression. This factor is incorporated in the following formula for the flash temperature at the pinion tooth tip.

Calculation of the flash temperature with the aid of a dimensionless parameter

By arranging the formulae for flash temperature in accord with the basic influential quantities such as specific loading w_{tw}, pitch line velocity $v_{tw},$ centre distance a, working pressure angle a_{tw} and a coefficient k characterizing the geometry of tooth contact, an insight is obtained into the effects of the measures to be taken in the design process. The speed and loading conditions in a gear pair can be related to an auxiliary quantity, i.e. a parameter of tooth contact. The definition of this tooth contact parameter k is illustrated by Figure 2, which represents the meshing conditions in any arbitrary in volute gear pair.

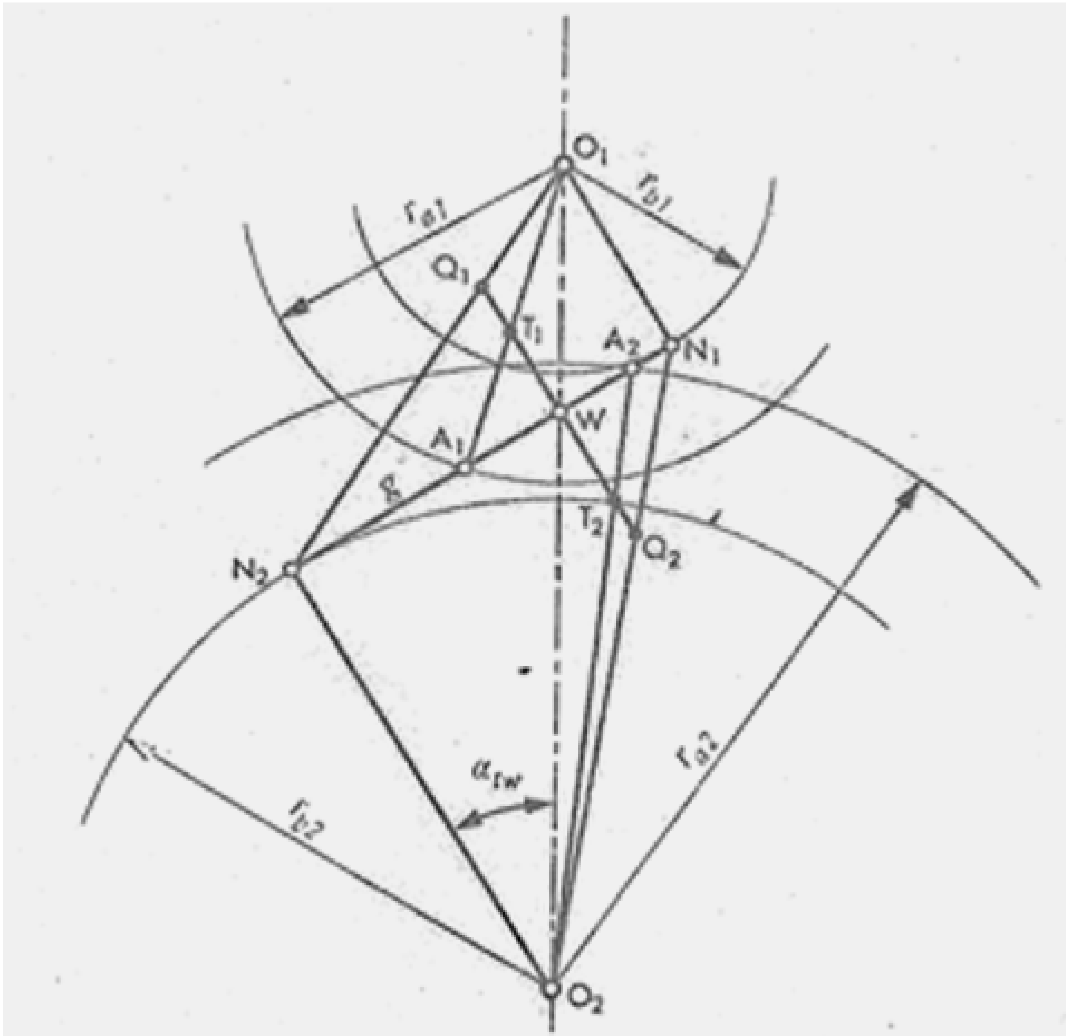


Figure 2: Tooth Contact Geometry in involute gears

0_1	Axis of pinion
0_2	Axis of wheel
W	Pitch point
g	Line of action
r_{b1}	Base circle radius of pinion
r_{b2}	Base circle radius of wheel
r_{a1}	Outside radius of pinion
r_{a2}	Outside radius of wheel
a_{tw}	Transverse working pressure angle

In Fig. 2, it can be shown that $Q_1W = Q_2W$

Tooth contact parameter for pinion addendum $k_1 = \frac{T_1\,W}{Q_1\,W}$

Tooth contact parameter for wheel addendum $k_2 = \frac{T_2\,W}{Q_2\,W}$

The formula for the tooth contact parameter for the pinion addendum is

$$k_1 = \frac{u+1}{u}\left[1 - \frac{t_g \propto_{tw}}{tg \propto_{a1}}\right]$$

where

$$\cos a_{a1} = \frac{d_{b1}}{d_{a1}}$$

And for the wheel addendum:

$$k_2 = [u+1] \cdot \left[1 - \frac{tg \propto_{tw}}{tg \propto_{a2}}\right]$$

where

$$\cos a_{a2} = \frac{d_{b2}}{d_{a2}}$$

With this parameter, expressions can be obtained systematically for the properties of a gear pair which are related to the sliding contact loading, such as the specific sliding δ (ratio of sliding velocity to rolling velocity) at any point of contact

$$\delta = \frac{v_s}{v_r} = \frac{k}{1-k}$$

Or to the ratio of the Hertzian pressure at any point to the value in the pitch point, or to the sliding velocity, or to the product PV, i.e. Hertzian pressure times sliding velocity.

PV is proportional to $\frac{k}{\sqrt{1-k}}$

It is moreover shown that the specific sliding at the extremities of the path of contact of a gear pair is balanced when $k_1 = k_2$. At the same time PV is also balanced[7]. With the aid of this parameter it is possible to systematize the tooth design of addendum modified gears[8].

There are a number of methods for calculating the flash temperature T_B at the tooth tips of pinion and wheel:

a) For electronic data processing on the basis of the fully written out formulae,
b) For accurate determination using simple computers, in the form of a basic formula containing the governing characteristics and two secondary values read from diagrams,
c) for an approximate assessment, by means of a characteristic load value for the scoring limit; this being read from a diagram.

<u>Method a):</u>

Formula

$$A_1 = \frac{\mathrm{w_{tw}} \cdot [v_{tw}]^{0.25}}{[z_1 \cdot z_2] \cdot a^{0.5} \cdot [\sin a_{tw}]^{1.25}}$$

Enables the flash temperature at the pinion tooth tip to be calculated from

$$T_{B1} = A_1 \cdot f_{10}$$

And at the wheel tooth tip from

$$T_{B2} = A_1 \cdot f_{20}$$

where

$$f_{10} = 60 \cdot \pi \qquad \frac{[u+1]^{0.25} \cdot {k_1}^{0.75} \cdot [u+1-u \cdot k_1]^{1.75} \cdot [u+1-k_2]}{u^{1.25}\,[1-k_1 + (1-k_1)^{0.5}\,] \cdot [k_1 + k_2 - k_1 \cdot k_2]}$$

$$f_{20} = 60 \cdot \pi \qquad \frac{[u+1]^{0.25} \cdot {k_1}^{0.75} \cdot [u+1-k_2]^{1.75} \cdot [u+1-u \cdot k_1]}{u^{1.5} \cdot [1-k_2+(1-k_2)^{0.5}] \cdot [k_1+k_2-k_1 \cdot k_2]}$$

For safety against scoring, the value of the following expression related to the primary governing characteristics

$$\frac{w_{tw} \cdot \sqrt[4]{v_{tw}}}{c_n \cdot c_R \cdot \sqrt{a}}$$

Must not exceed the value for C_{BP} read from the diagram; account must be taken of the pressure angle (a= 20˚ is assumed, see below figure).

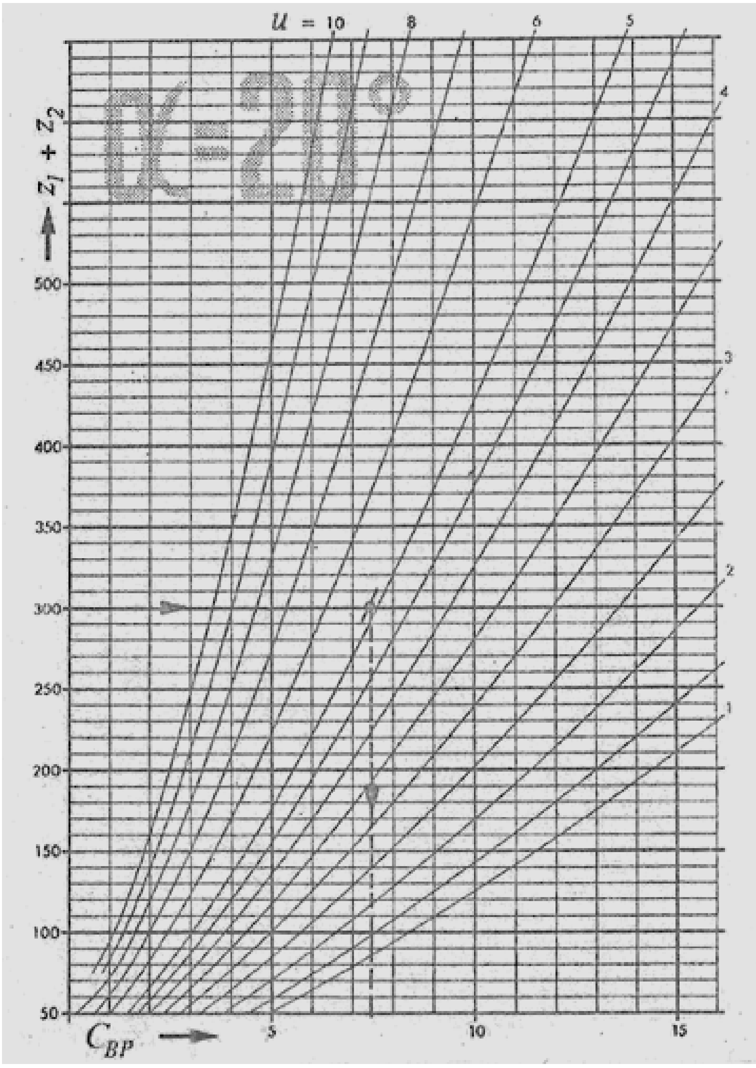


Diagram for the assessment of scoring risk: oil viscosity 4^0 Engler @ 50^{o}C

The scoring limit for the flash temperature

The permissible flash temperature T_{BP} depends on the properties of the lubrication oil, the surface finish of the tooth flanks and – since the formulae apply to error-free gears – also on the accuracy of the gear teeth and the assembly. The latter factors are likely to be typical for a given style of manufacture. Exact prediction of the scoring limit is therefore only possible for gears of like manufacture and on the basis of the relevant practical values observed. Even with one and the same manufacturing process there will be a certain amount of scatter. The permissible limit for the flash temperature therefore includes an adequate safety margin.

The permissible limiting value indicated below gives the scoring limit for gears not yet run in, or for gears without surface treatment after grinding, which must transmit full load immediately after starting up, using plain mineral oil devoid of any substantial additives. The calculated flash temperature T_B should be less than the permissible value $T_{BP.}$ Even in the event of equality there is an adequate safety margin within the manufacturing tolerance. In the case of gears subjected to more stringent demands, e.g. where a high rise in gear temperature is to be expected, or where a single gear meshes with several mating gears, it is advisable to provide for an extra safety margin.

Experience in the design of high capacity gear drives with hardened and ground gears suggest that the permissible flash temperature can be set at

$$T_{Bp} = 140 \cdot C_\eta \cdot C_R \text{ in}^\circ\text{C}$$

Where

Viscosity factor $C_\eta = \frac{75+R_{am}}{4\,R_{am}}$

E = viscosity factor $C_\eta = \frac{1.5 \cdot E}{2+E}$

Roughness factor $C_R = \sqrt{\frac{75+\ R_{am}}{4\,R_{am}}}$

Mean surface roughness $R_{am} = \frac{2\,R_{a1} \cdot R_{a2}}{R_{a1}+R_{a2}}$

R_{a1} = roughness of pinion flanks in $\mu\ ins$.
R_{a2} = roughness of wheel flanks in $\mu\ ins$.

Methods of raising the scoring limit

Electrolytic copper plating of the pinion with a coat thickness of 4 to 5 microns affords additional safety against scoring, for as this coat is worn away during operation the surface roughness is decreased. The use of oils with additives or synthetic oils raises the scoring limit considerably. Such measures must be discussed with the user beforehand, however, with regard to the acceptability and availability of such lubricant.

Conclusions

In the calculation of the load carrying capacity of gear pairs with respect to the third element participating in the transmission, i.e. the oil, the same practice can be applied as is familiar for determining tooth bending strength and resistance to surface loading. A relatively simple reference quantity or assessment characteristic can be calculated, and used as a datum for investigating the damage boundary by way of experiment. In use in practice, any conditions differing from those prevailing in the tests, or in other practical observations with regard to oil properties, surface finish, and load distribution during assembly etc. can be taken into account by way of coefficients. This process is familiar from tooth strength and surface stress calculations, where tooth form and load factors allow for the particular prevailing conditions.

The flash temperature criterion is a suitable elementary reference quantity for assessing the scoring resistance of the gears, since the influence of the oil viscosity and the surface roughness affects the coefficient of friction only. The assumption that this coefficient of friction can be expressed as a function of the surface loading and velocity of sliding accords well with practice over a wide loading range. Also important for a practical system of assessment of load carrying capacity is that the most adverse conditions are catered for with certainty, i.e. the conditions prevailing in gears before running in, lubricated with mineral oil without additives. The criterion is also adapted to the situation of high specific loading and pitch line velocities (over 100 m/sec). For lower specific loading the forecast is somewhat too pessimistic.

The value derived for the practical permissible flash temperature T_{PB} is, strictly speaking, only valid for a given manufacturing practice – in the case expounded, for hardened and ground tooth flanks with profile and longitudinal corrections, suitable bearings and careful assembly. The expression put forward contains a considerable, but not precisely known, safety margin, for it embraces the practical conditions of high capacity gear drive construction.

The suggested process for determining the scoring limit has been applied successfully in the relevant company's experience and in investigations into cases of defect has yielded explanations of the causes.

Expansion of this method into a universal process for determining oil film resistance will of course require further experiment, for example with the FZG-Test equipment for determining the individual influence of oil viscosity, surface roughness, and temperature rise (10). The ratio of Hertzian pressure to specific loading in thigh capacity gear drives is a little different from that in the above mentioned test, however. Tests are therefore also necessary on gears of larger dimensions.

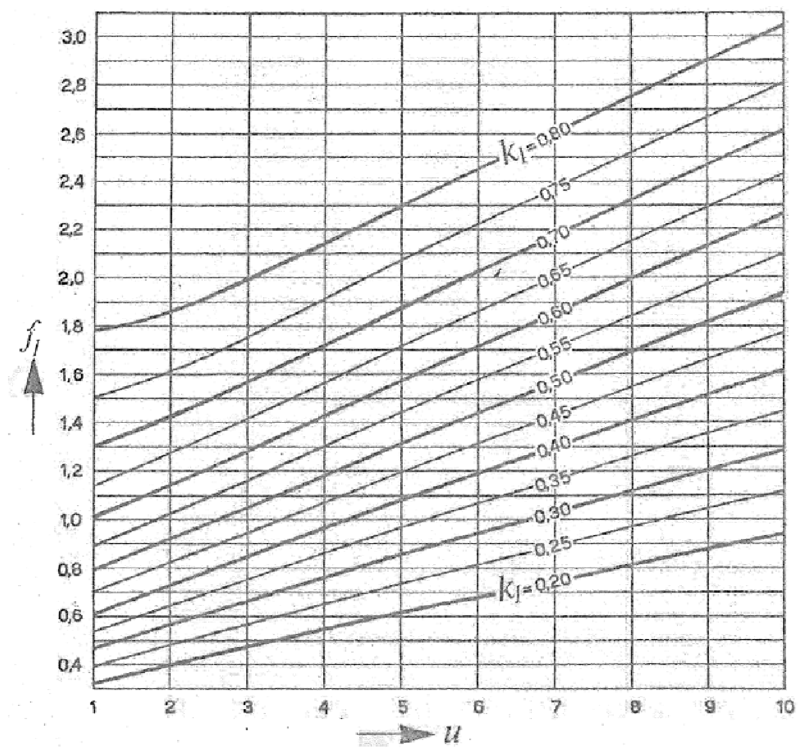
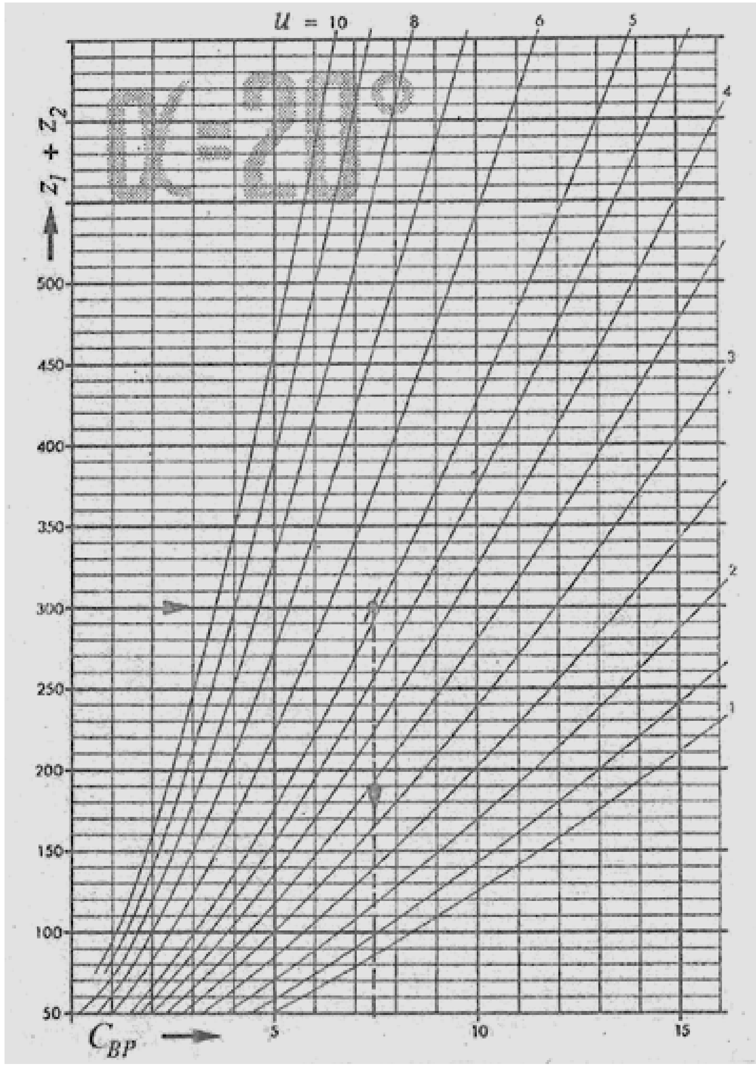


Fig. 3: The function $f_l = f(u, k_l)$



u = 10
8
6
5
4
3
2
1
α = 20°
z1 + z2
500
450
400
350
300
250
200
150
100
50
CBP
5
10
15

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