GEAR TOOTHING

by

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GEAR TOOTHING

In gear driven industrial plants the toothing occupies very little space. However, the successful operation of the plants and consequently the profits therefrom highly depend on this very important space. Successful plant operation depends on the optimum and most reliable transmission of the torque between the two gear rotors by the tooth flanks.

In this brief lecture we will concentrate on a few important topics concerning the involute gear toothing.

1 "MAAG Toothing"

"MAAG Toothing" is related to the tooth geometry and especially to the addendum modification (long addendum, addendum modification coefficient, rack shift) which was invented by Max Maag.

Two gears with involute tooth flanks have correct meshing conditions on any center distance if they have an identical base pitch and an identical base helix angle. The operating pitch circle of each gear does not have to be identical with the generating pitch circle (reference circle).

"MAAG Toothing" practically always has a center distance greater than half the sum of the reference diameters; in other words, the sum of the addendum modification coefficients of pinion and gear is always positive.

Figure 1

This represents the principle of the addendum modification $x \cdot m$ (x = addendum modification coefficient, m = module = 1/DP).

What are the advantages of this principle?
The amount of shifting of the basic rack simply defines nothing else than a certain part of the involute as active tooth flank. Without an addendum modification, the active flank may be too close to the base circle and the radius of curvature becomes very small, resulting in a high contact stress (pitting). Besides this, the relative sliding between the tooth

flanks of pinion and gear is greater and, consequently, the danger of scoring is increased.

With a favourable addendum modification these disadvantages can be avoided and an optimum tooth form with regard to tooth bending strength may, furthermore, be obtained (especially the fillet curve). The maximum possible addendum modification is determined by the minimum allowable tooth thickness at the tip.

Figure 2

This represents tooth form as a function of the number of teeth z, versus addendum modification coefficient x.

Figure 3

This approximate equation, which is very simple compared with the exact involute equation indicates the relation between center distance and normal module, number of teeth in pinion and gear, helix angle, sum of addendum modification coefficients.

The "MAAG Module Table" contains very fine steps. Because of this, a solution can always be found for this equation for a given center distance, a desired optimum module (with regard to bending and scoring), a sum of $(x_1 + x_2)$, a helix angle and a desired gear ratio z_2/z_1.

MAAG, consequently, applies this module table and there are no manufacturing difficulties, because low-cost rack-type cutters are used instead of expensive hobs. Tooth grinding machines and the tooth measuring machines do not depend on the module.

This was the "MAAG Toothing" at the time of Max MAAG.

The difficult problem still remained, however, of how to split the sum of the addendum modification coefficients into x_1 for the pinion and x_2 for the gear. 1958, Robert Wydler from MAAG published a calculating method based on non-dimensional contact parameters, to determine in each case exact values of the addendum modification coefficients of the pinion x_1 and of the gear x_2.

In 1972, he subsequently developed a flash temperature calculating method as a scoring criterion, which is also influenced by the addendum modification coefficients.

These methods allow for an optimum balancing of the gear meshing conditions, such as sliding versus rolling between the tooth flanks and also for the flash temperatures of pinion and gear.

In combination with profile and longitudinal modifications of the tooth flanks this is now the actual "MAAG Toothing", which is a systematical lay-out method, strictly applied day by day for any type of gear and is related to more than 20 years of sucessfull and reliable gear manufacturing.

2 Deformations of toothing and gear rotors
Profile and longitudinal modifications of involute tooth flanks

As already mentioned the most reliable transmission of the torque between the two gear rotors by the tooth flanks is of the greatest importance. Therefore, the mechanical deformations of the teeth and rotors and the thermal deformations of the rotors due to non-uniform temperature distribution have to be compensated for, by modifying the actual ground tooth flanks and thereby deviating from the mathematical true involute form, in order to obtain a satisfactory full load tooth bearing pattern.

2.1 Tooth load distribution along the path of contact in the transverse section

In order to prevent engagement shocks, due to tooth bending deformations, and to obtain a "trapeziform" tooth load distribution along the path of contact in the transverse section, profile modifications are always applied by means of tip and root reliefs on the pinion. The shape and magnitude of these modifications have to be carefully determined, in connection with the calculated tooth stiffness of the gear mesh, and with accumlated results in the field.

Figure 4

This shows the interference at the first point of contact (point A) of a true involute toothing without profile modifications. This interference is due to the bending deformations of the teeth number 1 and 2 being in contact to each other at point D. The resulting shock is a source of noise and it may decrease the strength of the toothing in all its aspects, including the oil film on the tooth flanks.

Figure 5

This shows the profile modification (profile diagram) of the pinion and the "trapeziform" tooth load distribution obtained along the path of contact without engagement shock.

2.2 Uniform tooth load distribution along the face width

In order to obtain a uniform tooth load distribution along the face width, a longitudinal modification on the pinion is always applied to compensate for the bending, torsional and thermal deformations of the gear mesh.

Figure 6

This represents the bending and torsional deformations of the pinion separately and combined, as well as the resulting longitudinal modification.

For gears with pitch line velocities above approx. 80 m/s (16000 ft/min) additional thermal deformations of pinion and gear have to be considered. These deformations originate from heat due to the power loss in the meshing zone by the displacement of the air/oil mixture out of the tooth spaces.

In a helical gear, this displacement moves along the face width (screw pump effect) from the cooler side to the warmer exit side. The side faces of the rotors, especially the larger one of the gear rotors have a cooling effect. The resulting thermal tooth flank deformation of pinion and gear has an asymmetrical barrel shape. Measured along the face width from the cooler side to the warmer side, the base tangent length would increase to a certain peak value and decreases toward the warmer side to a value which is still larger than the one on the cooler side.

If these thermal deformations are not compensated for, they cause a high local overload between the tooth flanks, seriously endangering the safe operation of the gear. To obtain an equal tooth load distribution along the face width, the pinion tooth flanks are provided with a conical/cocave longitudinal modification.

Very extensive tests performed by MAAG with pitch line velocities up to 200 m/s (39000 ft/min), using thermocouples installed in the gear rotors, cleary indicated that even with all kinds of asymmetrical oil injections into the gear mesh or gear housing, it is practically impossible to obtain a uniform temperature distribution in pinion and gear, which would allow for the omission of this conical/concave longitudinal modification.

The total longitudinal modification is the superposition of the modification for the mechanical deformations and the modification for the thermal deformations. The typical shape of such a longitudinal modification will be shown together with the data of an example of a high speed turbo gear, Fig.10.

Working flanks provided with such longitudinal modifications cannot be used to align a gear, because the blue tooth bearing pattern is much too short. Therefore, the non-working flanks of pinion and gear are ground absolutely parallel and are used as a basis for the correct alignment of the gear.

3 Calculation of load capacity

Many different calculating methods are applied around the world to calculate the load capacity of the gear toothing. Some of these are very simple and others are very detailed, for example:

MAAG ZURICH

MAAG Gearbook 1963 and 1985
AGMA Standards 421, 211, 221, 217
AGMA Standard 218
API Standard 613
ISO/DIN 6336 / 1 to 5 and 21
DIN 3990 / 1 to 5
A very simple method was the calculation of the "k factor" and of the "unit load" formerly used, a very detailed method is ISO 6336, part. 1 to 5.

Three criteria determine the load capacity:
a) surface durability or pitting (contact stress, k factor). Fig.7
b) strength (tooth root bending stress), Fig 7
c) scuffing or scoring (flash temperature)Fig.8

For the dimensioning of the toothing, the following two principles are generally applied:

- For the three criteria one defines design values of contact stress, tooth root bending stress, flash temperature depending on the application, materials and heat treatment process of gear rotors and on the lube oil employed.

  As these defined design values already contain large safety margins, which are based on the experience of the responsible gear manufacturer for each typical case of application, the introduction of safety factors or service factors is redundant.

- For the three criteria, minimum safety factors are defined by a responsible gear manufacturer for each type of application.
  A safety factor is calculated as the ratio between the permissible value of contact stress, tooth root bending stress, scuffing temperature and the corresponding effective value at operating conditions.

In our opinion it is not so important which one of all the existing calculating methods is used; however, it is important that, over a long period of time, these 3 criteria are always calculated with the <u>same</u> method, which is correlated to field experience. Only by following this procedure can design values for stresses or for flash temperatures or corresponding safety factors or service factors be developed for the satisfactory layout of a toothing for each typical case of application.

When one wants to switch from a present, simpler method to a new, detailed method, both methods must be applied in parallel over at least the next 5 years, so as to correlate them to actual experience and to be able to determine adequate safety factors for this new method.

A MAAG toothing always complies with the MAAG Gearbook 1963 and 1985 and with the MAAG flash temperature criterion of 1972. The corresponding allowable design values for contact stress, bending stress and flash temperature are related to the past 20 years of successful gear manufacturing. They are conservative and contain a large safety margin, but they do not necessarily oversize a gear.

If requested, a MAAG toothing additionally complies with the specified service factor of the corresponding standard, for example AGMA Standard 421.06 or API Standard 613.

In general the MAAG and AGMA philosophy go nicely together as long as the specified AGMA service factor is not unreasonably high (for example greater than 1.5).

The same value for the service factor using API Standard 613 results in a toothing with increased dimensions (face width, center distance).

For medium pitch line velocities up to 130 m/s (25000 ft/min) this oversizing of the gear may not do any harm, although it is not required for a MAAG toothing. At high pitch line velocities above 130 m/s (25000 ft/min) an oversized toothing would only produce considerably more power loss, larger mechanical deflections and thermal deformations of the tooth flanks, and, as a result, a <u>less</u> safe torque transmission between pinion and gear instead of additional safety. The authors of API Standard 613, therefore, included deviation paragraph 2.2.5, so the gear manufacturer is obliged to inform his customers about these conditions.

MAAG also applies the new detailed standards ISO/DIS 6336/1 to 5, which are published in the MAAG Gearbook 1985, a method here called "ISO-MAAG", which represents the application standard for all MAAG gears.

The intention is to establish a relationship between the MAAG method of MAAG Gearbook 1963, presently applied, and the new methods with regard to the required safety factors.

We already noticed that the minimum safety factors (1.3 for contact stress and 1.6 for bending stress) as indicated in the application standard DIN 3990 part 21 for high speed gears and gears of similar requirements are much too low for a gear design of high reliability.

Sufficient experience does not exist yet using the "flash temperature criterion" nor with the "integral temperature criterion". Both criteria are only valid for pitch line velocities up to 80 m/s (16000 ft/min). Therefore, at the present time, MAAG applies the MAAG flash temperature method of 1972, which is well proved for high speed turbo gears and gears of similar requirements with pitch line velocities well above 80 m/s (16000 ft/min).

4 Example of toothing of a high speed turbo gear

The toothing of a high speed turbo gear, a speed increasing gear between gas turbine and turbo compressor, shall be discussed as an example.

Specification:

Power	: 34590 kW (46400 HP)
Speed of gas turbine	: 4670 rpm
Speed of compressor	: 10500 to 10550 rpm
Lubricant	: Turbo oil ISO VG-32
Viscosity of lube oil	: 32 cST/40°C (150 SSU/104°F)
Lube oil inlet temperature	: 45°C (113°F)
Heat treatment of toothing	: carburized and case hardened
Surface hardness	: 58 HRC

For above conditions three different layouts for the toothing are calculated:

a) MAAG 1963 layout with MAAG design values for this specific application

b) AGMA 421.06 layout, with Service Factor = 1.6

c) API 613 layout, with Service Factor = 1.6

MAAG gear type GB-50, with a center distance of 500 mm (19.685 inch) is applied for the three layouts, MAAG, AGMA, API, resulting in a high pitch line velocity of 169.4 m/s (33350 ft/min).

Figures 9, 10, 11

In these figures, characteristic data of these layouts are put together for comparison. Some points of special interest shall be explained here:

4.1 Geometry of toothing

Each layout has an optimum combination, regarding number of teeth, normal module (normal diametral pitch), helix angle, center distance, addendum modification coefficients and actual speeds.

4.2 Face width

MAAG	AGMA	API
405 mm (15.945 in)	432 mm (17.008 in)	507 mm (19.961 in)

MAAG requires the smallest face width and the corresponding ratios of face width to pitch diameter are:

MAAG	AGMA	API
1.32	1.41	1.65

The ratio 1.65 of the API layout is too large, but a consequent increase of the center distance would raise the pitch line velocity above a desired design limit.

4.3 Load capacity: MAAG, AGMA, API

It can be clearly seen by comparing the acutal service factors, that MAAG and AGMA apply similar design philosophy.

The unnecessarily oversized API layout does not correspond with MAAG or AGMA.

In regard to the scoring criterion (flash temperature) the design values of MAAG and AGMA standard 217.01 are practically reached.

4.4 Power loss in gear mesh

MAAG	AGMA	API
320 kW	370 kW	440 kW
(430 HP)	(496 HP)	(590 HP)

The comparison of the three layouts clearly shows that MAAG, with the smallest face width and the smallest module (largest diametral pitch), has the smallest power loss in the toothing.

The higher the power loss is, the higher the heat that has to be taken out of the gearbox by the lube oil and the larger the thermal deformations of the gear rotors. The combination of the highest power loss with the largest face width (API layout) would only create an uncertainty in obtaining a uniform tooth load distribution along the face width, even with a longitudinal modification.

Instead of having a gear with increased safety, the gear would actually have a reduced reliability.

4.5 Mechanical and thermal deformations, longitudinal modifications

Figure 10

This illustrates the above mentioned problems in more detail.

The mechanical deformation of the pinion, due to bending and torsion, the combined thermal deformation of pinion and gear and the resulting total deformation between the tooth flanks of pinion and gear are indicated for the MAAG layout. The thermal deformation is of much higher magnitude than the mechanical deformation.

The corresponding longitudinal modifications are drawn to scale for the MAAG layout, the AGMA layout and the API layout, at the bottom of Fig. 10.

4.6 Load capacity: ISO/DIS 6336 1 to 5

In order to illustrate the relation between the calculating methods MAAG 1963 and the new ISO standards, the safety factors in accordance with ISO/DIS 6336 1 to 5 are determined for the MAAG toothing layout.

In the calculations an application factor K_A = 1.25 is considered.

Figure 11

This shows the actual safety factors determined for two cases:

A) toothing provided with proper longitudinal modification,
(face load factor = 1.1)
safety factor for durability S_H = 2.4
safety factor for strength S_F = 3.4

B) toothing without longitudinal modification,
(face load factor = 1.7)
safety factor for durability S_H = 1.9
safety factor for strength S_F = 2.2

The striking difference between the safety factors of case A) and B) makes it obvious that proper longitudinal modifications must be applied by all means!

5 Final Remarks

To conclude this brief lecture about gear toothing, let me state the following:

Despite all the applied calculating methods, standards, the help of computers and numerical controlled machine tools, the gear toothing still remains a work of art. Satisfactory results essentially depend on the know-how of the people involved and above all on their will to do their best every day. This is mainly true for the staff of the heat treatment, gear cutting, gear grinding and gear inspection departments, but is also true for the assembly shop and for those out in the field.

Gears from different gear manufacturers may follow identical safety or service factors, determined in accordance with official standards to calculate load capacity, but their reliability can nevertheless turn out to be totally different - due to the disparity in the quality of workmanship. Even if a gear is properly designed and manufactured, it might still suffer damage, if it is put in service without the correct alignment.

Accordingly, every possible effort has to be made in relation to the gear toothing. One has to see to it that foreseen savings will not turn out to endanger the design philosophy or be responsible for lowering the accuracy. This could be disastrous in regard to the reliability of the plant and thus cause loss of production.

A gear customer is well advised to select a gear manufacturer that he can trust as much as he trusts his own dentist.

Gentlemen, MAAG hopes that you always select the best "dentist"!

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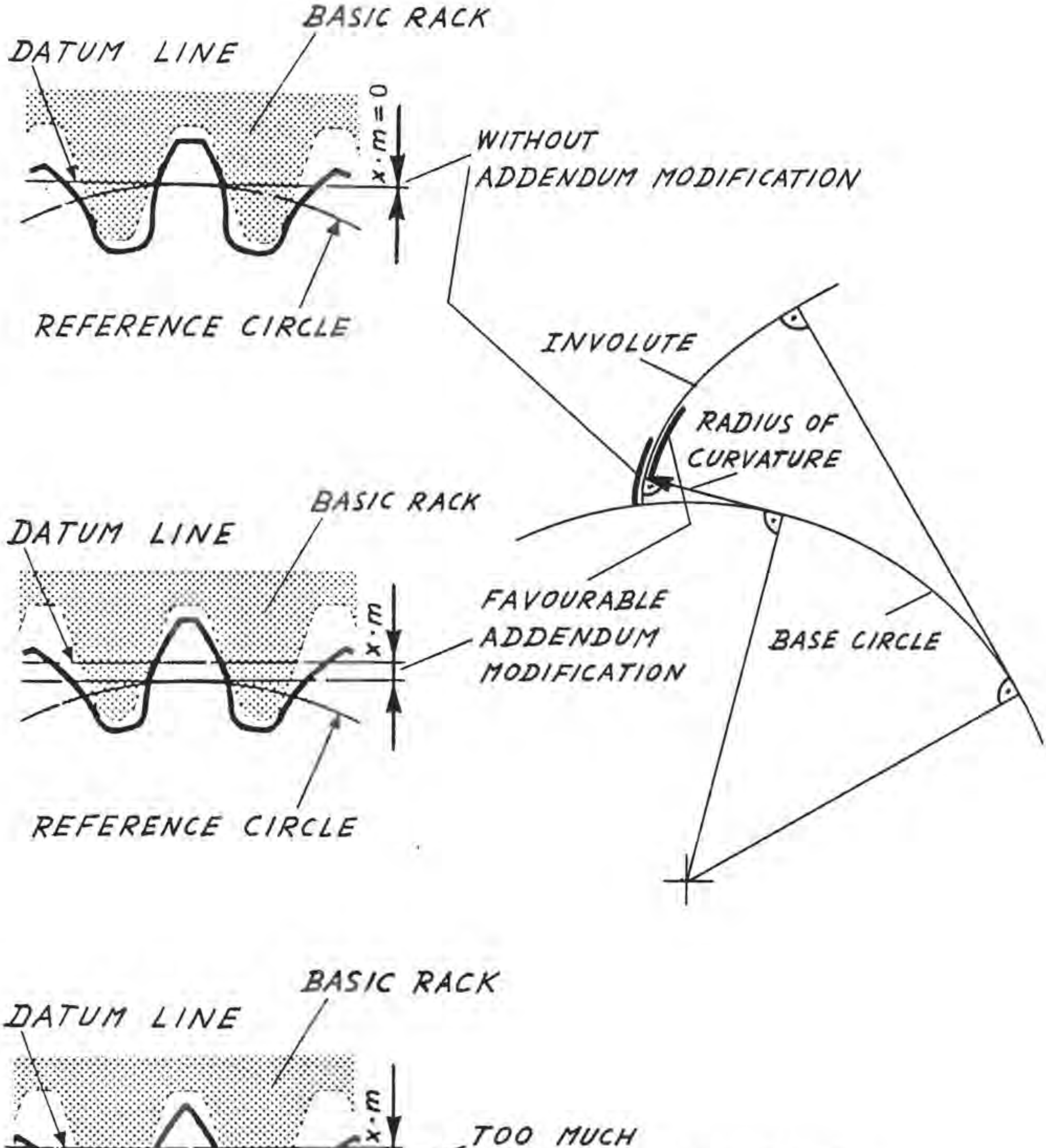
BASIC RACK
DATUM LINE
x·m=0
WITHOUT ADDENDUM MODIFICATION
REFERENCE CIRCLE
INVOLUTE
RADIUS OF CURVATURE
BASIC RACK
DATUM LINE
x·m
FAVOURABLE ADDENDUM MODIFICATION
BASE CIRCLE
REFERENCE CIRCLE
BASIC RACK
DATUM LINE
x·m
TOO MUCH ADDENDUM MODIFICATION
REFERENCE CIRCLE

FIG. 1

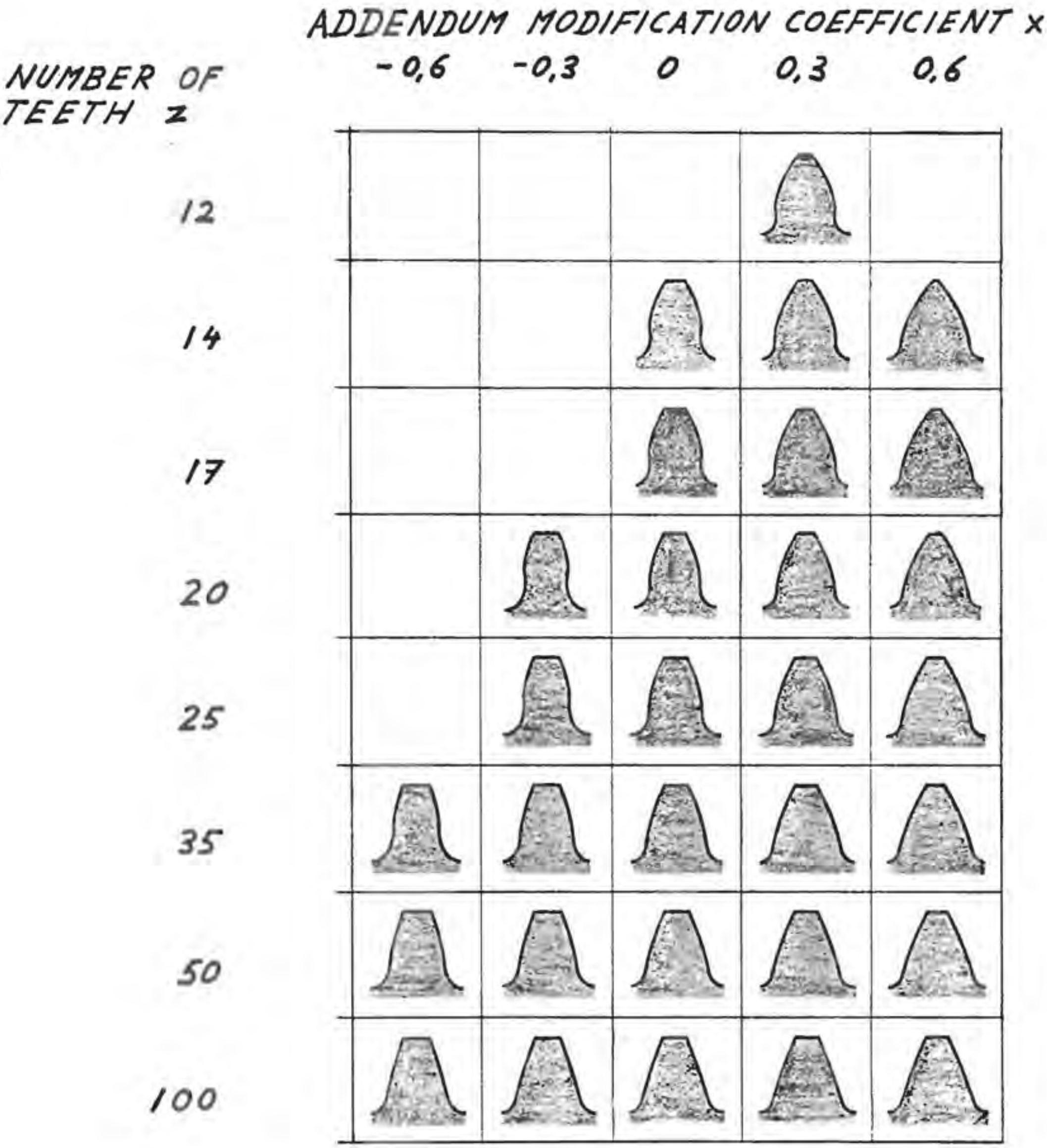
ADDENDUM MODIFICATION COEFFICIENT x
-0,6
-0,3
0
0,3
0,6
NUMBER OF TEETH z
12
14
17
20
25
35
50
100

FIG. 2

$$a \approx m_n \left[\frac{z_1 + z_2}{2 \cdot \cos\beta} + (x_1 + x_2) \right]$$

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a = CENTER DISTANCE
m_n = NORMAL MODULE = $1/DP$
z_1 = NUMBER OF TEETH IN PINION
z_2 = NUMBER OF TEETH IN GEAR
β = HELIX ANGLE
x_1 = ADDENDUM MODIFICATION COEFFICIENT OF PINION
x_2 = ADDENDUM MODIFICATION COEFFICIENT OF GEAR

$$(x_1 + x_2) = 0 \div \underline{0.3 \div 0.8} \div 1.3$$

FIG. 3

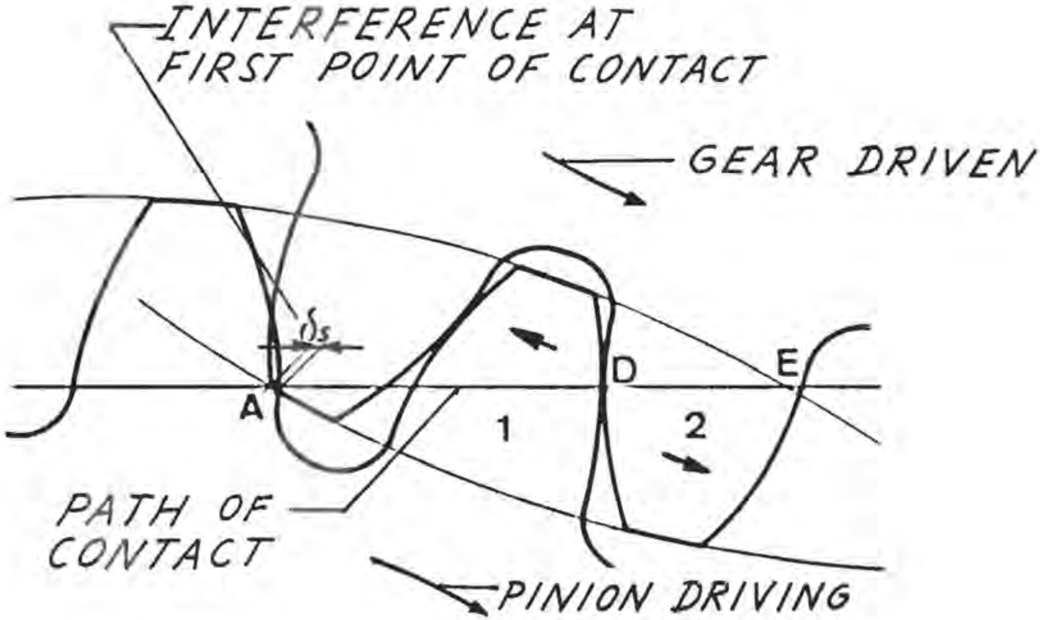
INTERFERENCE AT
FIRST POINT OF CONTACT
GEAR DRIVEN
δ_s
A
D
E
1
2
PATH OF
CONTACT
PINION DRIVING

FIG. 4

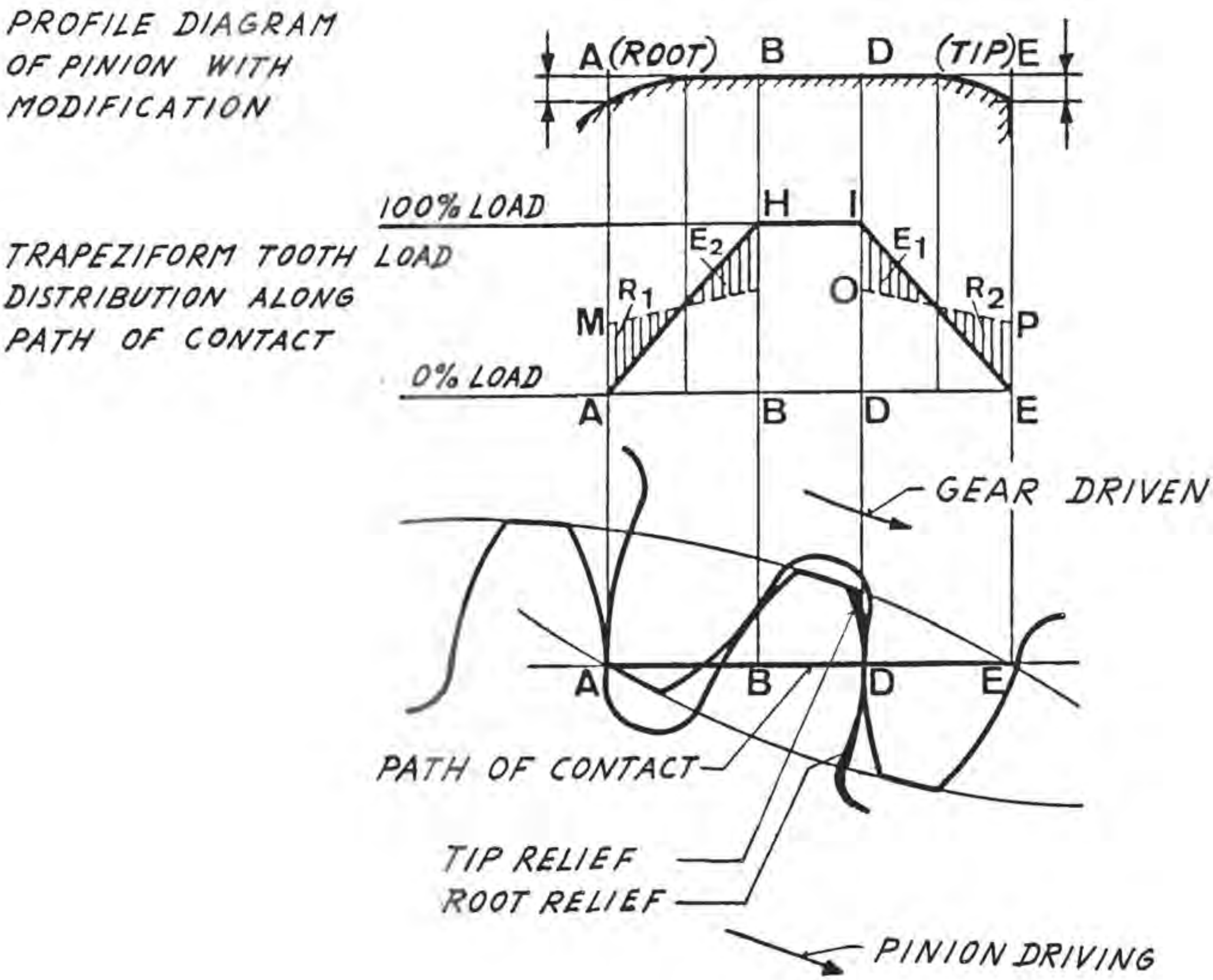
PROFILE DIAGRAM OF PINION WITH MODIFICATION
A (ROOT)
B
D
(TIP) E
100% LOAD
H
I
TRAPEZIFORM TOOTH LOAD DISTRIBUTION ALONG PATH OF CONTACT
E2
E1
R1
R2
M
O
P
0% LOAD
A
B
D
E
GEAR DRIVEN
A
B
D
E
PATH OF CONTACT
TIP RELIEF
ROOT RELIEF
PINION DRIVING

FIG. 5

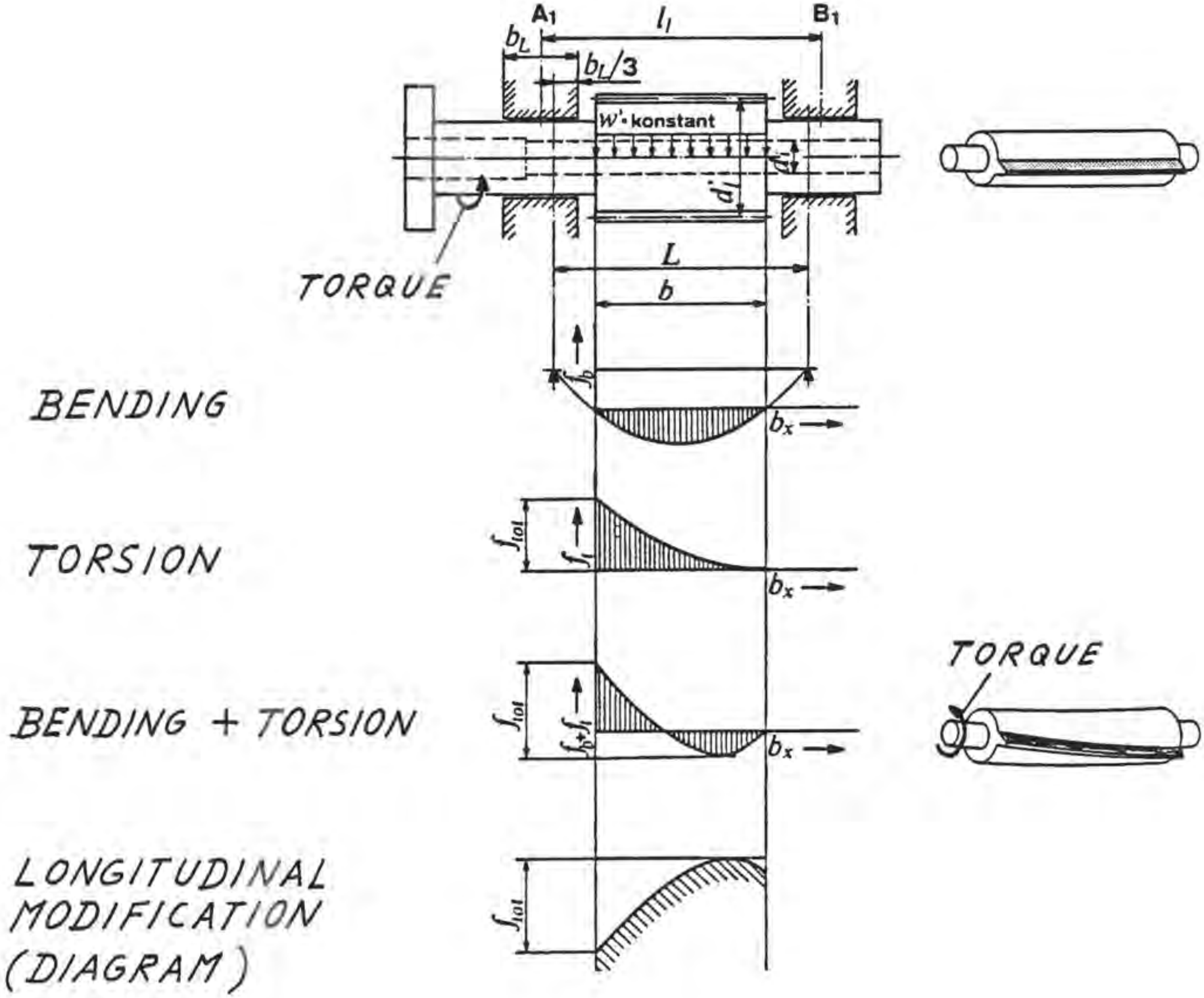
A1
l_1
B1
b_L
$b_L/3$
w'·konstant
d_1
d_1'
TORQUE
L
b
BENDING
f_b
b_x
TORSION
f_{tot}
f_t
b_x
BENDING + TORSION
f_{tot}
f_b+f_t
b_x
TORQUE
LONGITUDINAL
MODIFICATION
(DIAGRAM)
f_{tot}

FIG. 6

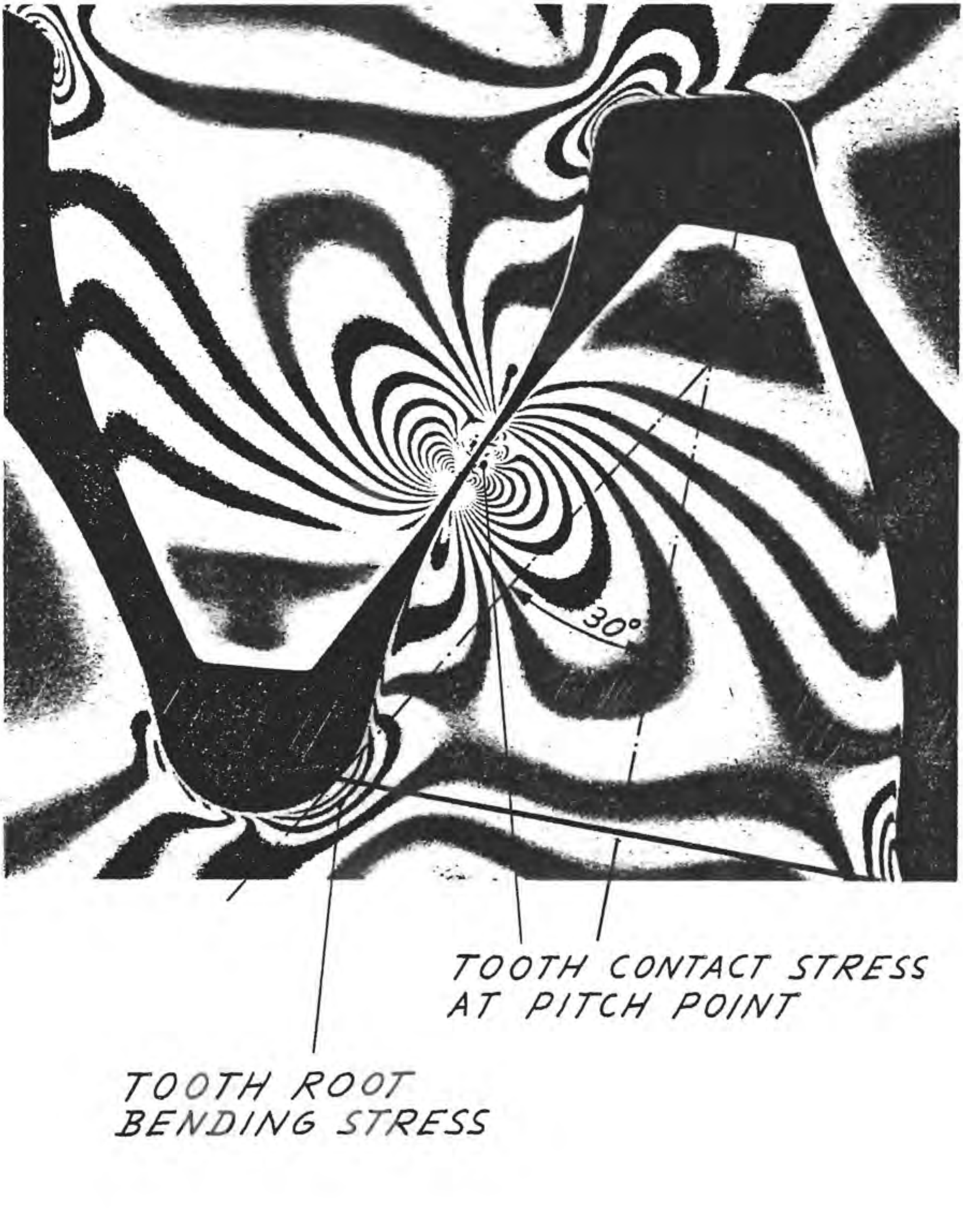
30°
TOOTH CONTACT STRESS
AT PITCH POINT
TOOTH ROOT
BENDING STRESS

FIG. 7

SCORING STREAKS

HEAVY SCORING

SCORING ZONES

SCORING (SCUFFING)

FIG. 8

TURBO GEAR: GAS TURBINE / COMPRESSOR DRIVE 34590 kW (46 400 HP) AT 4670/10500 $^{+0,5\%}_{0}$ RPM				
LAYOUT OF TOOTHING		MAAG 1963	AGMA 421	API 613
DESCRIPTION	UNITS			
CENTER DISTANCE	mm (in.)	500 (19.685)	500 (19.685)	500 (19.685)
NUMBER OF TEETH	—	106/47	97/43	88/39
NORMAL MODULE	mm	6.30	6.90	7.625
NORMAL DP	(in^{-1})	(4.0317)	(3.6812)	(3.3311)
FACE WIDTH	mm (in.)	405 (15.945)	432 (17.008)	507 (19.961)
PITCH LINE VELOCITY	m/s (ft/min)	169.4 (33350)	169.4 (33 350)	169.4 (33 350)
LOSS IN GEAR MESH	kW (HP)	320 (430)	370 (496)	440 (590)
LOAD CAPACITY:				
MAAG 1963 / 1972				
- CONTACT STRESS	N/mm²	695	674	627
- BENDING STRESS	N/mm²	113	99	78
- FLASH TEMPERATURE	°C	135	135	125
AGMA 421 / 217				
- DURABILITY, CSF	—	1.70	1.77	2.0 *
- STRENGTH, KSF	—	1.42	1.64	2.10*
- FLASH TEMPERATURE	°F	243	248	246
API 613				
- PITTING, SF	—	1.28	1.37	1.60*
- BENDING, SF	—	1.13	1.30	1.66*

* ONLY APPARENT SERVICE FACTOR

FIG. 9

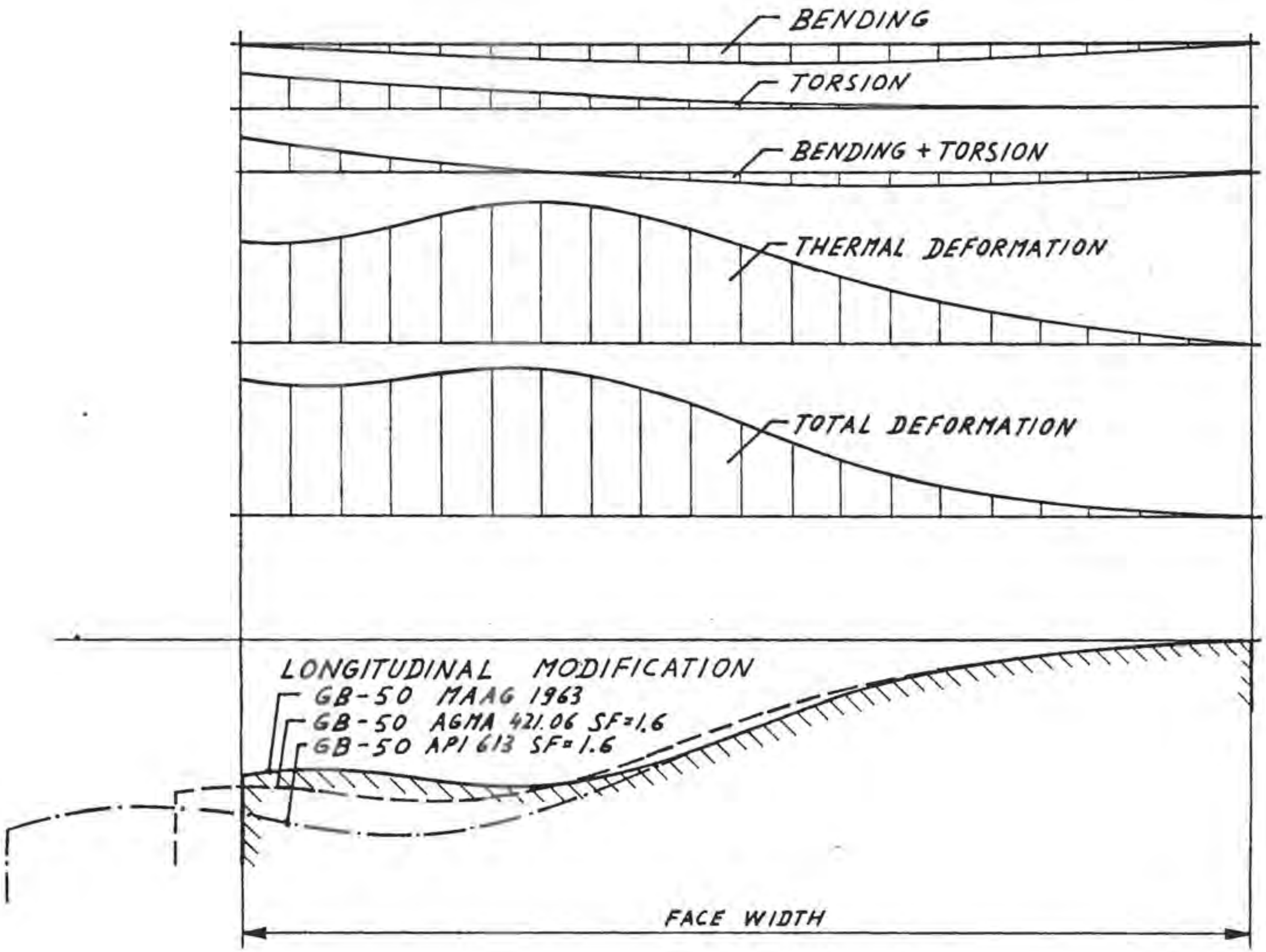
BENDING
TORSION
BENDING + TORSION
THERMAL DEFORMATION
TOTAL DEFORMATION
LONGITUDINAL MODIFICATION
GB-50 MAAG 1963
GB-50 AGMA 421.06 SF=1.6
GB-50 API 613 SF=1.6
FACE WIDTH

FIG. 10

TURBO GEAR: GAS TURBINE / COMPRESSOR DRIVE
34590 kW (46400 HP) AT 4670/10532 RPM

CALCULATION OF LOAD CAPACITY (LAYOUT MAAG 1963)
ISO/DIS 6336 / 1 ÷ 5

CASE A: WITH LONGITUDINAL MODIFICATION
FACE LOAD FACTOR $K_{H\beta}$ = 1.1
APPLICATION FACTOR K_A = 1.25

CASE B: WITHOUT LONGITUDINAL MODIFICATION
FACE LOAD FACTOR $K_{H\beta}$ = 1.7
APPLICATION FACTOR K_A = 1.25

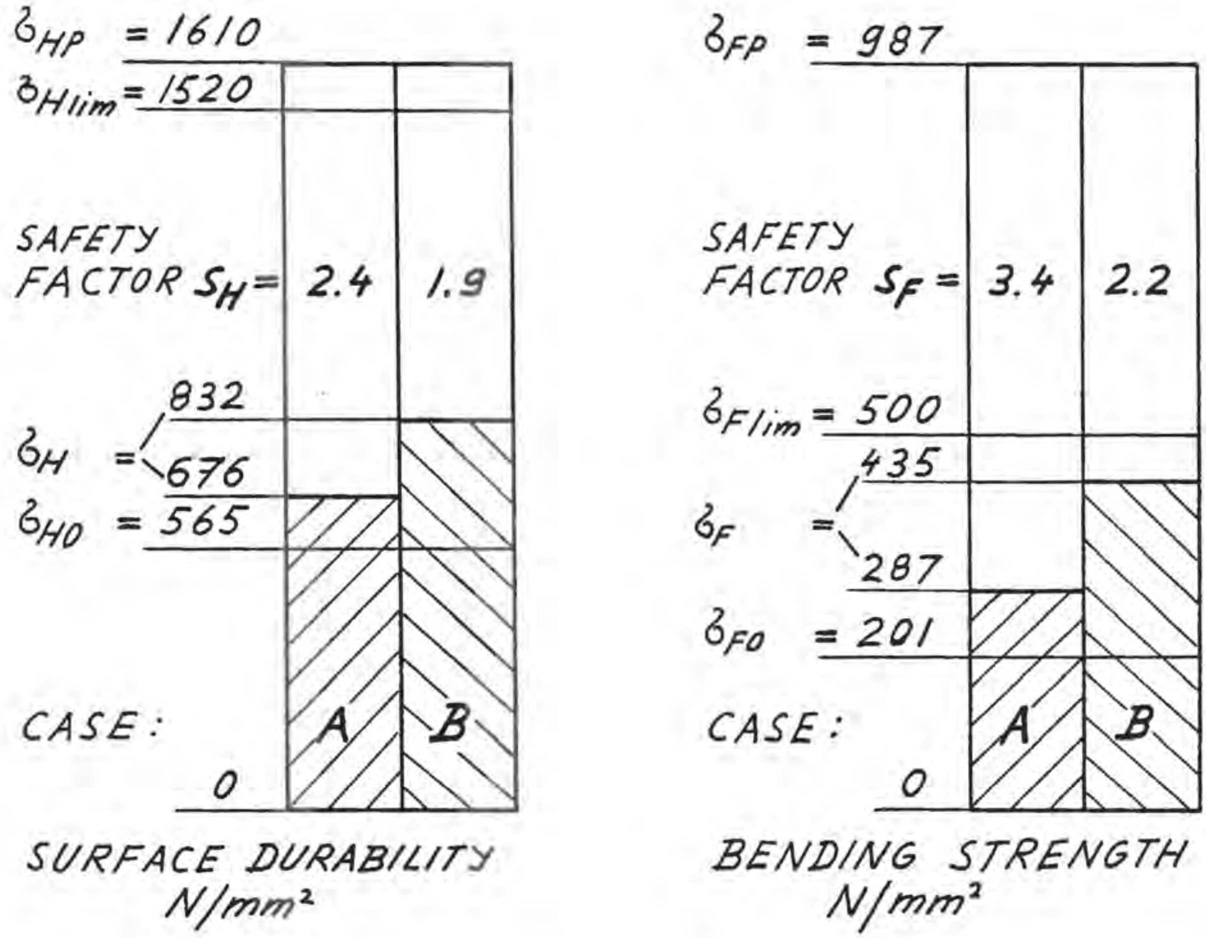

FIG. 11