A PROCEDURE FOR PREDICTING THE LOAD DISTRIBUTION AND TRANSMISSION ERROR CHARACTERISTICS OF DOUBLE HELICAL GEARS

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ABSTRACT:

A contact analysis algorithm based on linear programming is used for predicting the load distribution and transmission error characteristics of double helical gears. The analysis accounts for the axial shifting of the double helical pinion that enables the loads to be shared equally between the two halves of the double helical gear. Inclusion of the axial shifting of the pinion is shown to be crucial to the accurate determination of the load distribution along the lines of contact. The effect of the axial stiffness of couplings upon the amount of axial shift predicted for the pinion is presented. The effect on transmission error of stagger between the teeth of the two halves of a double helical gear shows that a suitable choice of stagger can minimize transmission error.

INTRODUCTION

This paper presents the development of an analytical method for evaluating the load distribution and transmission error in double helical gears. The programming procedure proposed by Conry and Seireg [1] for the analysis of elastic bodies in contact forms the basis of the method developed for the solution of this problem. This procedure has been successfully utilized in the past for developing the Load Distribution Program (LDP) [2] for the analysis of load distribution and transmission error in spur and helical gears. The actual solution of the contact problem in double helical gears is obtained by using the simplex type procedure proposed by Vijayakar, et al. [3], for the solution of multibody frictional contact problems.

In this paper, the two helical gears of opposite hands of the helix angle constituting the double helical gear will often be referred to as the two halves of the double helical gear. In order to facilitate equal load sharing between the two halves of a double helical gear, the general design practice is to allow the pinion to float axially while the position of the gear is fixed.

A literature review revealed that little work has been published in developing procedures for analyzing double helical gears and that most of the existing design procedures are of a purely empirical nature making little distinction, if any, between the design of double helical gears and single helical gears. No literature could be found on the analysis of transmission error characteristics of double helical gears. Rockwood, et al. [4], presented a method for the analysis of double helical gears using finite element analysis in conjunction with the method proposed by Conry and Seireg [1]. However, their procedure for analysis does not take into account the axial shift of the double helical pinion.

When analyzing double helical gears with the LDP program, the conventional approach has been to consider one half of the double helical gear and perform the analysis without accounting for the presence of the other half. Another approach has been to model the double helical gear as a single helical gear having its face width equal to the sum of the face widths of the two halves. It is evident that these approaches will supply, at best, rough approximations of the actual load distribution characteristics of double helical gears.

The method proposed herein, simultaneously analyzes the two helical gears of opposite hand using their geometry data on an 'as is' basis and accounts for the effects resulting from the axial shift of the pinion. Hence, it is felt that the proposed method gives a more accurate and realistic picture of the load distribution and transmission error characteristics of double helical gears.

FORMULATION OF THE SOLUTION PROCEDURE

Double helical gears are used in applications where the axial thrust component of the load on single helical gears is excessive. Axial thrust reduction is achieved by enabling the load to be shared equally between the two halves of the double helical gear, thereby causing the net axial component of the loads on the respective halves to be equal and opposite. It should be noted that equal load sharing between the two halves of a double helical gear does not imply that the load is necessarily distributed symmetrically over the two halves and only means that the net load on each of the halves is equal.

The usual design practice to achieve equal load sharing between the two halves is to permit the pinion to float axially in its bearings [5]. There may be resistance to axial float resulting from the friction in the shaft couplings. If there is enough resistance to axial movement, desired equalized load sharing between the two halves may be impaired. Dudley [5] has stated that a 30° helix angle or higher is needed in the double helical design to ensure that the resistance to axial movement from the coupling is minimized. Fig. 1 shows the model of a double helical pinion where the pinion is allowed to float freely in the axial direction but is resisted by the axial stiffness of a coupling.

In the analysis of double helical gears a variable that defines the amount by which the pinion shifts axially and a constraint equation that forces the loads to be equal in the two halves of the double helical gears is necessary. When the pinion shifts axially, it causes an offset in each of the gear teeth in mesh. The effect of this offset is to change the initial separation between the potential points of contact on the two halves of the double helical gears. If the pinion of the gear pair in mesh is a right hand helical gear, then the effect of the axial shift of the pinion to the right is to cause an increase in the initial separation between the potential points of contact on this gear pair and if the pinion of the gear pair in mesh is a left hand the reverse shift occurs.

One must also consider the frequency with which the pinion changes its axial position during a mesh cycle. Three different approaches are used in this analysis of double helical gears with each approach differing only in the manner in which it treats the axial shift of the pinion.

The *first approach* is to allow completely free axial motion of the pinion, thus causing the pinion to have a different axial position at each position in the mesh cycle. This is a reasonable assumption so long as the factors inhibiting axial shift, namely, high friction of the shaft couplings and large inertias of the gears are not large enough to warrant their inclusion in the analysis

The *second approach* is based upon the assumption that the individual or cumulative effects of the friction between the coupling devices and the inertia of the gears become significant enough to prevent continual motion of the pinion in the axial direction. This is a very logical assumption when mesh frequency effects are the primary consideration, since the forces required to axially move the gear inertias at this frequency are usually very large, resulting in negligible axial motion. This can be easily checked by assuming the motion to be sinusoidal and applying Newton's law as axial force, $F_a = mx'' = -m\omega^2 x$ where ω is the mesh frequency, m is the mass of either gear, and x is the axial displacement. With this approach, one assumes that while meshing, the pinion always occupies the position of mean axial shift obtained from the analysis using the first approach and continues to operate at that position.

The *third approach* combines the first and the second approaches with the difference being the incorporation of the resistance of the pinion to axial shift due to all factors in the form of an axial stiffness specified by the user. With this approach, for the segment of the analysis that computes the mean axial position, the difference in the axial thrust components of the two halves of the double helical gear at any position in the mesh cycle is set equal to the coupling axial force. Keeping the above discussion in mind, the three analysis approaches are detailed below:

Analysis considering unrestrained axial motion of the pinion

For this approach, δ_a, the variable defining axial shift, can have a different value for each position in the mesh cycle and is not constrained by any considerations of the system's inability to shift freely in the axial direction. To ensure the existence of a mathematical solution of the load distribution problem, the following conditions must be satisfied:

(a) Condition of compatibility: "For contact to take place at any point k in the zone of contact, the sum of the total elastic deformations of the two bodies and the initial separation between the bodies must be greater than or equal to the difference of the rigid body approach along the line of action and the change in separation at that point due to the axial shift of the body as a whole [1]."

This is mathematically stated as:

$$W_k^{(1)} + W_k^{(2)} + \varepsilon_k \geq R_b\theta - \delta_a \tan\psi \cos\phi_t \quad (1)$$

where,

$W_k^{(1)}$ is the total elastic deformation of point k on body 1.

$W_k^{(2)}$ is the total elastic deformation of point k on body 2.

ε_k is the initial separation at point k.

R_b is the base radius.

θ is the rigid body rotation along the line of action.

δ_a is the rigid body axial shift of the pinion.

ϕ_t is the transverse pressure angle of the gears.

ψ is the helix angle of the half of the double helical gear on which the point lies.

Writing this condition for all of the points in the zone of contact, we have

$$[S]\,[F] + [\varepsilon] \geq [e]\,R_b\theta + [K]\,\delta_a \quad (2)$$

$[S] =$ N×N matrix of influence coefficients.

$N =$ $N_L + N_R$ where N_L is the number of the points of contact on the left half of the double helical gear and N_R is the number of points of contact on the right half.

$[F]_L =$ N_L×1 vector of the forces at left half contact points.

$[F]_R =$ N_R×1 vector of the forces at right half contact points.

$$[F] = \begin{bmatrix}[F]_L \\ [F]_R\end{bmatrix}$$ is the N×1 force vector.

$[K] =$ N×1 vector containing $\tan\psi\cos\phi_t$ in its first N_L rows and $\tan\psi\cos\phi_t$ in its next N_R rows.

$[\varepsilon] =$ N × 1 vector of initial separations.

$[e] =$ N × 1 vector of ones.

(b) Condition of equilibrium: This condition warrants that the total moment about the axis of rotation of the forces $F^{(k)}$ acting along the line of action is equal to the applied torque, T. Mathematically, this condition specifies that:

$$[F]^T [e]\,R_b = T \quad (3)$$

(c) Condition for contact: This condition states that the two bodies must be in contact at a point for a pressure to exist at that point. Rewriting equation (1) as an equality constraint by introducing a slack variable Y_k, we have:

$$W_k^{(1)} + W_k^{(2)} + \varepsilon_k - R_b\theta + \delta_a \tan\psi\cos\phi_t - Y_k = 0$$

where $Y_k \geq 0$

The condition for contact implies in mathematical terms that at any point k in the zone of contact,

$$\text{If } Y_k = 0 \text{, then } F^{(k)} \geq 0$$
$$\text{If } Y_k > 0 \text{, then } F^{(k)} = 0 \tag{4}$$

(d) Condition for good design: This condition requires that the net force carried by the two halves of the double helical gear be equal. This is mathematically expressed as:

$$\Sigma\ [F]_L = \Sigma\ [F]_R$$

In computing the elastic deformations, we assume that a force acting at any point of any tooth on either gear pair in the double helical arrangement will influence the shaft bending and torsional deflections as well as bearing deflections at any other point on either gear pair. However, the same force will influence the tooth bending at another point only if that other point is on the same tooth of the same gear pair and within the range of influence of the applied load. Thus, the influence coefficient matrix for shaft and bearing deformations will be well filled with respect to the influence coefficient matrix for tooth bending effects. The influence coefficient matrix for Hertzian deformation will be a diagonal matrix. Thus, the formulation of the load distribution problem may be stated as:

Find the values of (F, Y, θ, δ_a) subject to the following constraints:

$$-[S]\ [F] + R_b\ [e]\ \theta + [K]\ \delta_a + [Y] = [\mathcal{E}]$$

$$[F]^T\ [e]\ R_b = T$$

$$\Sigma\ [F]_L = \Sigma\ [F]_R$$

either $F^{(k)} = 0$ or $Y_k = 0$ where,

$[Y] = N \times 1$ vector containing the slack variables.and all other variables are as defined previously.

The problem thus stated is solved using the Simplex type procedure of Vijayakar, et al. [3], to obtain the solution for the load distribution problem.

<u>Analysis considering the pinion to operate at the position of mean axial shift</u>

From the solution of the problem as formulated for unrestrained axial motion of the pinion, the amount of axial shift of the pinion (δ_a) for each mesh position is obtained. However, as stated before, when the friction and axial stiffness of the shaft couplings are high and/or the inertia of the pinion is large, it is likely that the pinion will reach a mean axial position and will remain in this position. Thus the load distribution problem needs to be analyzed one more time, now with the position of the pinion fixed at its mean axial position "δ_m" (shown in Fig. 3). δ_m is obtained from the expression:

$$\delta_m = \frac{\sum_{i=1}^{N} [\delta_a]_i}{N}$$

where N is the number of positions in the mesh cycle.

Hence, δ_a is no longer a variable in the equations. With the removal of one variable, one must also remove the associated constraint equation that requires forces the forces carried by each of the two halves of the gear to be equal. Now, the average loads on the two halves are equal for one mesh cycle.

The total initial separation between the potential points of contact is computed by adding the change in initial separations associated with the mean axial position of the pinion to the initial separations computed from the geometry of the gears. The analysis is then carried out by solving for (F, θ, Y) from the following constraint equations:

$$-[S]\ [F] + R_b\ [e]\ \theta + [Y] = [\mathcal{E}_N]$$

$[F]^T\ [e]\ R_b = T$ where

$$[\mathcal{E}_N] = [\mathcal{E}] - \delta_m\ [K]$$

$\delta_m =$ the mean axial shift of the pinion as computed previously.

<u>Analysis considering the resistance to axial motion of the pinion</u>

As mentioned earlier, this approach is a combination of the first two approaches with a change that the resistance to axial motion of the pinion is included in the form of the specification of the coupling axial stiffness. Referring to Fig. 1, the resistance to axial motion of the pinion is incorporated into the analysis as follows:

The 'condition for good design' which is specified in the first approach as:

$\Sigma\ [F]_L = \Sigma\ [F]_R$ now becomes:

$$\Sigma[F]_L \cos\phi_t \tan\psi = \Sigma[F]_R \cos\phi_t \tan\psi + K_a \delta_a \tag{5}$$

where K_a is the axial stiffness specified by the user. All other terms are as defined previously, noting that multiplying a force acting in the transverse plane in the direction of the line of action at a point on the tooth of a gear by the factor "$\cos\phi_t \tan\psi$" will give the component of that force in the direction of the axis of the gears.

This implies that in the limiting case where the axial stiffness tends to zero, the results from this analysis will be the same as obtained from the first approach. In the case where the axial stiffness becomes large, the difference between the load carried by the left and the right halves of the double helical gears will increase, as shown in equation 5. This is in accordance with what one expects to occur in the physical system, where large resistance to axial motion limits the amount of axial shift and thereby impairs equal load sharing between the two halves of the double helical gear.

Thus, in this approach, the amount of axial shift for each position in the mesh cycle is computed by solving for (F, θ, δ_a, Y) from the following constraint equations:

$$-[S]\,[F] + R_b\,[e]\,\theta + [K]\,\delta_a + [Y] = [\mathcal{E}]$$

$$[F]^T\,[e]\,R_b = T$$

$$\Sigma[F]_L\cos\phi_t\tan\psi = \Sigma[F]_R\cos\phi_t\tan\psi + K_a\,\delta_a$$

From the axial shift obtained for the various positions in the mesh cycle, one can compute the mean axial position at which the pinion operates. Having obtained the mean axial position, the remainder of the analysis is carried out exactly as detailed in the second approach, giving the predicted load distribution and transmission error characteristics of the double helical gear pair.

RESULTS

The cases presented use the same double helical gear geometry shown in Tabled 1. However, variations exist in the different cases with respect to the non-gear geometry parameters of the system such as the inclusion or non-inclusion of the effects due to shaft deflections, axial stiffnesses of the couplings, and misalignment between the bearings. Cases were also run for equivalent single helical gears having face widths of 50 mm and 100 mm. These cases will be discussed without graphical outputs for brevity. (The double helical gear had 50 mm face width per half).

The results presented are classified into three different cases that are detailed below:

Case A: Study of transmission error considering the effects of tooth deflection only

The objective of this case study was to determine the difference in the transmission error characteristics of double helical gears and single helical gears when considering the effects of tooth deflection alone, i.e., when all shaft and deflections are excluded from the analysis.

The transmission error curve obtained for the double helical gear is shown in Fig. 2. The equivalent 50 mm face width single helical gear had identical transmission error characteristics but the equivalent 100 mm face width single helical gear differed in its behavior with its peak-to-peak transmission error (PPTE) value being only about 40% of that of the double helical gear as shown in Table 2. This occurs even though the double helical gear has twice the total contact ratio as the single helical gear with the 50 mm face width and the same contact ratio as the one with 100 mm face width. This is because the manner in which the load sharing between the teeth changes in each mesh cycle is the same for both the double helical gear and the single helical gear with 50 mm face width. When one tooth is leaving contact on one half of the double helical gear, a corresponding tooth is also leaving contact on the other half and thus the increased contact ratio of the double helical gears does not result in more gradual changes in the amount of load being carried by individual teeth. On the other hand the single helical gear with 100 mm face width has the full benefit of a more gradual change in the load being carried by it, which results in smaller mesh stiffness variation and lower transmission errors. Therefore, from a purely transmission error point of view, it is more advantageous to use a single helical gear with the same net face width than the double helical gear.

Further studies with the same gear sets as above, while including shaft bending, shaft torsion and bearing deflections showed varied results for each gear set indicating that it is not prudent to analyze a double helical gear by analyzing equivalent single helical gear model.

Case B: Study of the results from the different double helical gear models

Earlier it was discussed that there could be three different approaches for dealing with the phenomenon of the axial shifting of the double helical pinion in the analysis. In this case study the results obtained from the three different approaches are discussed. Also discussed is the result obtained when the double helical pinion is allowed no axial shift.

Using the first approach, the axial position of the pinion relative to the no load position is evaluated at 21 different positions in the mesh cycle. This gives the axial motion curve shown in Fig. 3. Using this data, the mean axial position of the pinion relative to the no load position is computed to be

10.9 μ m. which is used to perform the analysis using the second approach.

The next step in this case study is to determine the magnitude of the axial stiffness which causes a substantial reduction in the magnitude of the mean axial shift. Using the third approach the mean axial position of the pinion relative to its no load position was determined at values of axial stiffness varying from 175 N/mm to 175×10^6 N/mm. The results are plotted in Fig. 4 where it can be seen that the change in the magnitude of the mean axial shift relative to the no load position is small for axial stiffnesses of the order of magnitude less than or equal to typical values of bearing stiffness (200×10^3 N/mm) and decreases substantially for very large values of the axial stiffness. This implies that the axial shifting of the pinion will not be affected by typical values of the coupling axial stiffness which are usually at least an order of magnitude less than bearing stiffnesses. An axial stiffness of 350×10^3 N/mm gives a change in axial position of the pinion relative to its no load position of 6.6 μm, a value which is used in the subsequent analysis using the third approach.

The transmission error curves of the double helical gear obtained from the analysis using the three different approaches were almost identical. Table 3 shows the PPTE value for each case. This data leads to the conclusion that the amount of axial shift has little effect upon the transmission error characteristics of the double helical gear.

The load distribution along the lines of contact for each case was also studied (sample output shown in Fig. 5). It was seen that while the load distribution pattern along the individual lines of contact on the corresponding halves of the double helical gear are essentially the same for each approach, there remains a significant difference in the amount of load being carried by each half. In the ideal case, where the pinion is allowed to shift freely in the axial direction, the load being carried by each half of the double helical gear is identical, as expected. In the second approach the pinion is constrained to operate at its mean axial position and, hence, there arises a slight difference in the amount of load being carried by each half of the double helical gear. In the third approach, where the amount of axial shift is further reduced due to the axial stiffness prescribed by the user, the difference in the load carried by each half is larger. In the final case where no axial shift of the pinion is allowed, the difference in the loads being carried by each half becomes significant (30% difference between sides).

Case C: Study of the effects of stagger of the teeth

Sometimes double helical gear teeth of the two halves are staggered relative to one another. It is the objective of this case study to ascertain the benefits of staggering the teeth with respect to the minimization of transmission error.

In the computer program, the user is asked to input the stagger between the teeth in the form of a percentage stagger which is defined as the amount of stagger as a percentage of the base pitch. Hence defining a 50% stagger would imply that when viewed in the transverse plane, a tooth of one half of the double helical gear would be placed symmetrically between two teeth of the other half.

The double helical gear used in the previous case studies was analyzed for 10%, 20%, 30%, 40% and 50% stagger between the teeth while considering only the tooth deflection effects in the analysis. Percent stagger above 50% gives mirror image results to stagger below 50%. It was observed that with an appropriate stagger, one could achieve more than 50% reduction in the PPTE of the double helical gear (refer to Table 4). The variation of the total length of contact in the mesh cycle was also studied for each case. The transmission error curve for the case with 20% stagger is shown in Fig. 6 and the variation of the total length of contact lines in the mesh cycle for the same case is shown in Fig. 7. It is seen that the least PPTE was obtained for the case where there is 30% stagger of the teeth where the variation of the total contact length in the mesh cycle takes place more smoothly than in any other case. The greatest PPTE is seen in the case with no stagger where the variation of the total contact length in the mesh cycle is also the least smooth. It can thereby be concluded that by staggering the teeth, it is possible to achieve better transmission error characteristics for the double helical gear by causing smoother variation in the total length of contact in the mesh cycle. However, one must keep in mind that with the inclusion of shaft effects in the analysis, the optimum amount of stagger which provides the best reduction in the PPTE may differ.

SUMMARY

This paper has presented a procedure for analyzing the load distribution and transmission error of double helical gears for several situations. It was found that the analysis of single helical cases are not adequate for double helical gears and that the introduction of stagger between the teeth of each half can reduce transmission error.

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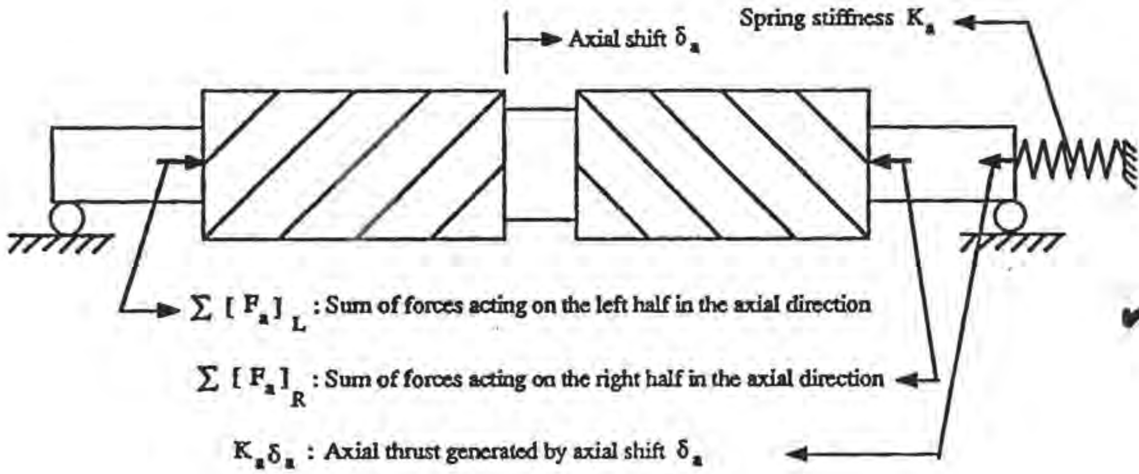


Fig. 1 Double Helical Gear Model Incorporating Axial Stiffness

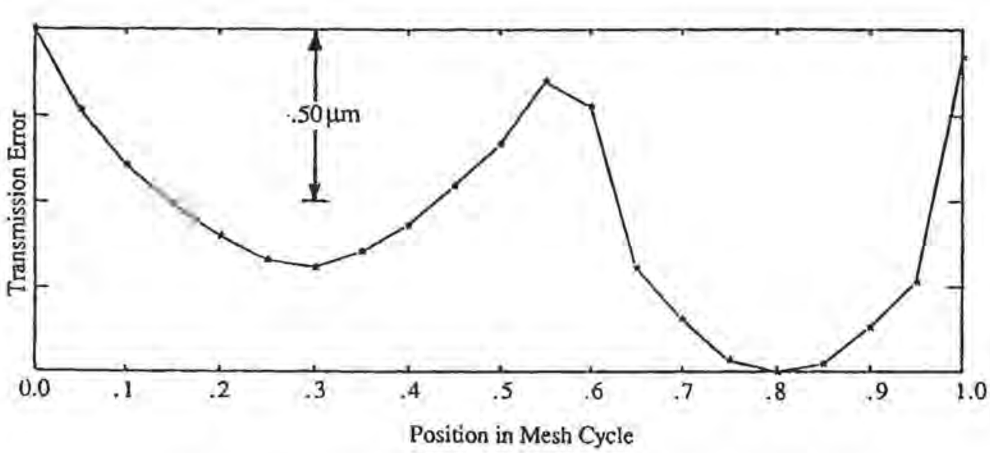


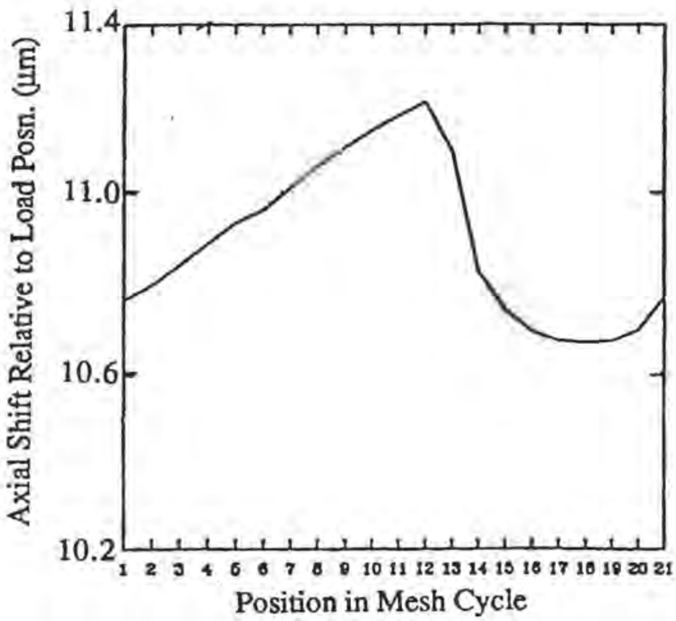
Fig. 2 Transmission Error for the Double Helical Gear Pair

Table 1 Gear Geometry

	Pinion	Gear
Number of Teeth	13	127
Outside Diameter	75.2mm	615.3mm
Root Diameter	53.7mm	594mm
Center Distance	338mm	
Normal Pressure Angle	20 °	
Module	4.11mm	
Helix Angle	31°	
Face Width	50.8mm	

Table 2 Transmission Error Comparison for Different Gear Types

	PPTE (μ m)
Double Helical Gear (50 mm FW / half)	1.05
Single Helical Gear (50 mm FW)	1.05
Single Helical Gear (100 mm FW)	.42



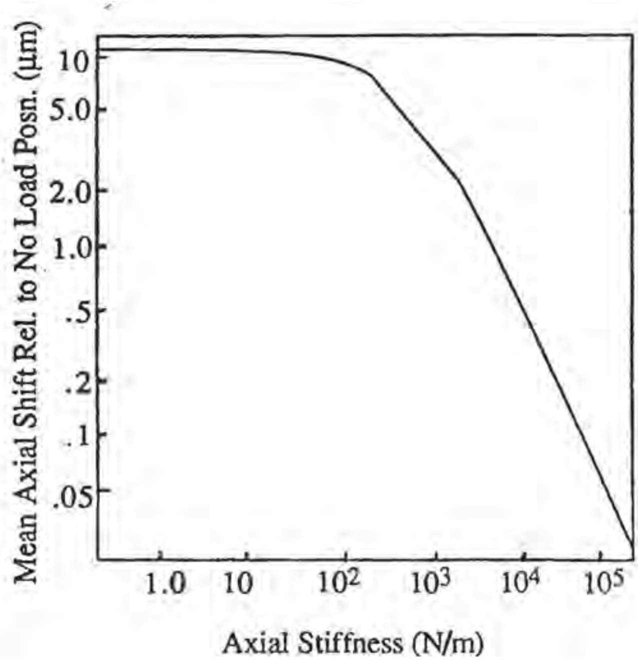


Table 3 Transmission Error Comparisons for Different Approaches

Analytical Procedure	PPTE (μ m)
First Approach	3.302
Second Approach	3.310
Third Approach	3.297
No Axial Shift	3.276

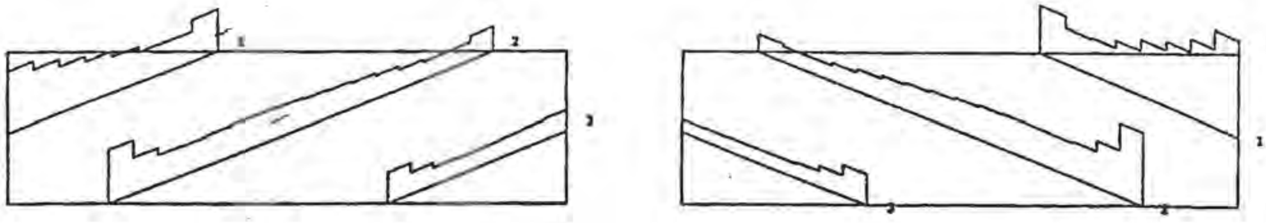


Fig. 5 Load Distribution Along the Lines of Contact

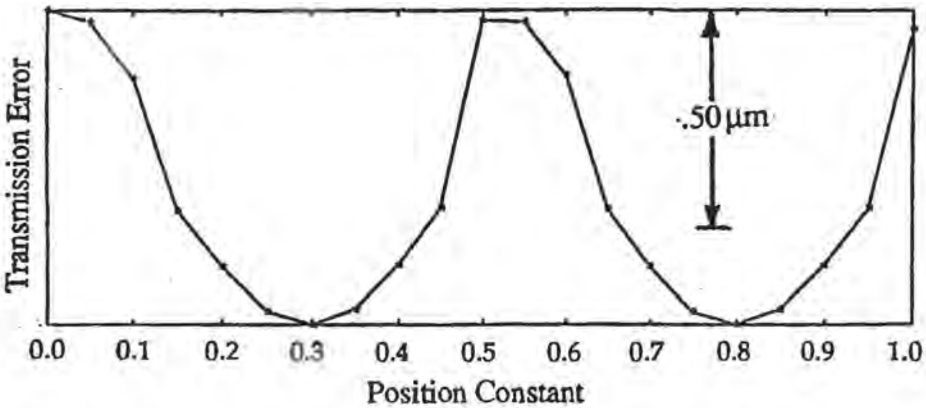


Fig. 6. Transmission Error over One Mesh Cycle for 50% Stagger

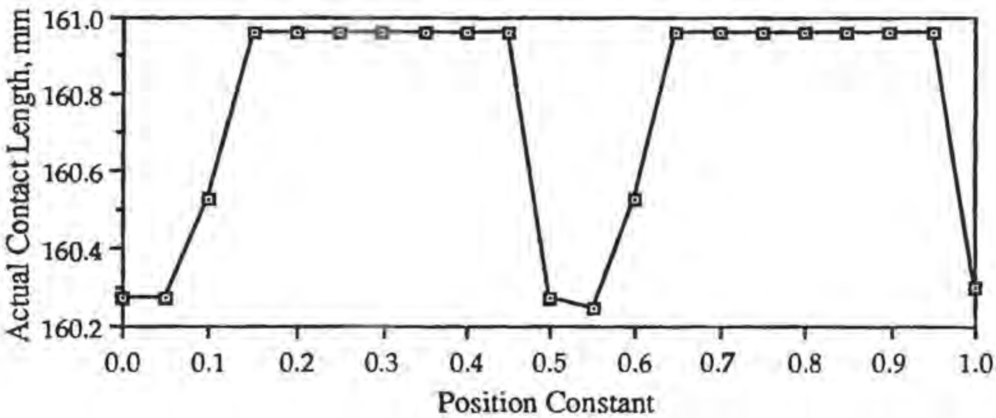


Fig. 7 Variation of Total Contact Line Length Over One Mesh Cycle.

Table 4 Transmission Error at Different Staggers

% Stagger	PPTE (micron)	Amplitude of the harmonics of the Fourier series of the TE curve(μ m)		
		Harm. #1	Harm #2	Harm #3
0%	1.05	.154	.391	.051
10%	.811	.149	.280	.026
20%	.578	.129	.108	.171
30%	.413	.095	.106	.046
40%	.719	.050	.280	.039
50%	.712	.001	.347	.003