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PROFILE AND LONGITUDINAL CORRECTIONS ON INVOLUTE GEARS

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Introduction

Experiments on loaded gear drives show that as tooth pairs move into and out of the field of mesh, shocks arise which cause fluctuations in angular velocities. Such shocks would occur even in perfectly accurately made gears, for they are partly due to the elastic deflection of the teeth. The strength of the shocks is dependent on the actual load being transmitted, and on the accuracy both of the teeth and of the mutual location of the gears in their housing. Other factors such as pitch line velocity, the moment of inertia of the gears, the surface quality of the flanks, and lubrication etc., will also exert some influence.

These fluctuations of the gears produce noise by inducing vibrations in the gears themselves but also in the shafts and housing.

When gear manufacture was still in its infancy, the general aim was to produce teeth as near to the theoretical form as possible. It was not until demands for higher speeds and loading, together with quieter running, became really acute, that the prospect of lessening engagement shocks by way of profile correction (tip and root relief) was entertained. As soon as the grinding of hardened gears was introduced, still higher specific loads could be transmitted which made the application of profile corrections even a necessity.

But with the increase in transmission loads, longitudinal corrections (or crowning) gained in significance also. As will be explained later in more detail, well designed crowning in connection with helical gears can also contribute to a reduction of engagement shocks, although the main object of crowning is to attain uniform load distribution across the facewidth under a given load; in other words, to counteract those various influences which are alien to good tooth bearing conditions.

So the considerations governing the application of the two types of flank correction – profile or longitudinal – are different. And for this reason they will be dealt with separately in this paper.

The actual degree of correction necessary, whether it is tip, root or end relief, is generally relatively small – usually between about 3 tenthousandths and 1 thousandth of an inch. But in spite of being so small, these corrections improve the load carrying capacity of a given gearing appreciably[1] providing they are designed and applied properly. However, if a tooth correction is to increase load capacity, it is only logical that a certain minimum manufacturing accuracy must be assured. In a case where tooth errors verge on the profile correction in order of magnitude, an improvement of the meshing conditions would be in doubt; particularly if error and corrections are additive.

It may generally be claimed that the certainty of improved loading capacity by way of profile and longitudinal corrections is only present if tooth errors are less in magnitude than the degree of correction.

The recommendations made in this paper are based almost exclusively upon experience gained from hardened and ground single helical gears. The tooth accuracy of ground toothings corresponds to an AGMA quality 14 to 15. The profile accuracy, however, is even of a better quality.

1. Basic Considerations

A gear tooth moves into the field of mesh with such a high speed, that the nature of its load take-over will have the character of a damped vibration. In spur gears, the number of teeth carrying the load changes from two to one and back to two, which makes the elastic reaction even more complicated. In helical gears there are more tooth pairs in contact and the effect of the changing number is less acute, although the situation is basically the same. For identical load, speed and tooth accuracy, corrections may be smaller on helical teeth than on spur teeth. A further consideration is that a helical tooth does not make contact immediately with its full facewidth. The load is taken up first at the leading end of the helix, and spreads across the whole tooth gradually (Fig. 1). For this reason, a longitudinal correction (crowning or end relief) will also be effective in avoiding engagement shocks. In the following we shall examine the conditions prevailing in meshing spur gears, from the purely static point of view. It must be kept in mind th the term "engagement shock" refers to the dynamic process of meshing, and that the actual force will exceed the theoretic, static value, and assume a vibrational form governed by the speed and inertia of the gears.

2. Tooth Loading Characteristics on True-Involute Spur Gears

When spur gears mesh, contact is made by one pai and two pairs of teeth alternately. Taking the line of action for an abscissa, as in Fig. 2, we can represent the force on the tooth at any point along the path of contact AD by an ordinate perpendicular to this axis. Two pairs of teeth make contact over the portions AB and CD, one pair only over the portion BC. The actual lengths of these paths are given by the gear dimensions, AC and BD being equal to the base pitch. For absolutely accurate, non-elastic gears, the load in the double contact regions would be exactly half of that in the single contact region; this is shown by curve AFGHIKLD. Due to surface deformation at the points of tooth contact, and to shear and bending deflection of the teeth themselves, the division of load alters. From calculations one obtains a force curve AMNHIOPD. Roughly speaking, engagement starts at A with 40% load, rising to 6 at the point of transition from two-tooth contact to one-tooth. After carrying 100% load alone over the central region, the receding tooth pair is attributed 60%, sinking to 40% again on moving out of the field of mesh.

3. Preventing Engagement Shocks

As soon as the teeth under consideration have some sort of error, the loading characteristic will be different again; especially since gear teeth are comparatively stiff, and even slight errors have a great effect. Of particular interest in our present investigation is the rotation of one gear relative to the other as a result of the elastic deflection of the teeth. We can express this rotation as a displacement δ_s along the line of action (see Fig. 3). Its value for spur gears would be:[2]

$$\delta_s = \text{approx. } 5 \times 10^{-3}\, w_g$$
in tenthousandths of an inch (Equation 1)

where w_g = the normal force along the line of action in lbs/in.

At the moment of moving into engagement, a driven tooth Z_{g0} will find its mating profile shifted along the line of action by the stated amount δ_s in accordance with Fig. 3; this is the result of the elastic deflection of the tooth pair Z_{g1} and Z_{p1} already in contact. Such lack of correlation will lead to engagement shocks. As already mentioned, tooth errors can have a similar effect, since they also represent contact point displacement.

Before devices for grinding flank corrections were incorporated in MAAG machines, the following practices were employed to aid the situation when manufacturing high power, high speed gears:

a) Close tolerances, particularly for profile form and adjacent pitch
b) Highest possible transverse contact ratio (one such measure was the introduction of addendum modification, based on a 15° cutter pressure angle – known since 1908 as MAAG-toothing)
c) Slightly longer base pitch for the driving teeth as compared to the driven teeth, as shown in Fig. 4.

Provided that the difference between the base pitches of the driving and driven gears is greater than the sum of all errors and deflections, the tooth tip of the driven gear sweeps into the field of engagement without making contact. The load is subsequently taken up by this tooth gradually. Naturally the difference must not be too great, or the ratio of the base circle diameters will no longer agree with the transmission ratio, and new speed fluctuations will be induced in frequency with the consecutive tooth engagement. In practice a base pitch difference of maximum 1.5 tenthousandths may be allowed.

By using helical gears with adequate overlap ratio (say between 3 and 4), the influence of tooth errors, especially of profile errors is less felt. There is also less tendency for base pitch differences to cause speed fluctuation.

4. Purpose and Principles of Profile Correction

To avoid shocks as gear teeth enter and leave the field of engagement, the flank profile can be eased back locally over a suitable distance – in the regions of the tip and root on the pinion for instance, familiar to all as "tip and root relief". Nowadays the form and degree of such corrections can be controlled accurately on MAAG grinding machines, and moreover the relieved areas can be blended smoothly into the remaining true-involute areas.[4]

Various aspects dictate the character of the correction to be made; and those for spur gears differ from those for helical gears, so that different sets of corrective principles evolve. The final verdict concerning the worth of a given correction can only be passed on the strength of practical results.

To examine the problem as presented by spur gears, we shall refer again to the diagram of load in Fig. 2. We see that on true involute flanks, quite apart from initial and final contact in A and D, there is an abrupt change in loading at the change points B and C, where the load is suddenly transferred from two teeth to one tooth alone and vice versa. Since this can excite vibrations, such shocks must be suppressed as far as possible. In Fig. 5b an assumed specific loading is delineated, which should afford some success in this respect.

Neglecting manufacturing errors for the moment, we are faced with the question: What exact form must the correction take, in order to make the force of tooth contact follow the graph AHID in Fig. 5b instead of the graph AMNHIOPD which would apply if the flanks are not corrected?

In Fig. 5a the tip of a driven tooth is just making initial contact in A. Another tooth pair already makes contact in C. Just before point C, the full load is carried by the one tooth pair, causing the point of contact to be displaced along the line of action by the amount δ_s as per equation 1. If the newly contacting tooth tip is left uncorrected, the tooth would immediately take up a load represented in Fig. 5b by the point M. By easing back the profile of the said tip an amount equal to δ_s, the load is reduced as desired from M to zero. The relief must finish at contact point B_1. Field R_1 in Fig. 5b represents the actual load of which the newly contacting tooth pair is relieved. Since the total load must remain unchanged, the preceding tooth pair already in contact is subjected to a correspondingly higher load, represented by field E_1. The appropriate geometric relief is plotted to an enlarged scale along the line of action in Fig. 5c, giving a diagram comparable to that obtained from a tooth profile recording instrument.

On the tooth tip itself the correction will appear as shown, highly enlarged, in Fig. 5a. By easing back the tip of the receding, driving tooth tip, the load on the latter is reduced in a like manner as it moves out of mesh. Applying practically the same geometric tip relief, we achieve a load reduction R_2 and a load increase E_2. It is by such tip corrections on driving and driven flanks that the contact forces are made to follow the graph AHID of Fig. 5b. As can be seen, there are no abrupt load changes. Along the path B_1C_2 transmission takes place via true-involute flanks. Distance B_1C_2 is equal to the base pitch. Based on these observations we arrive at the following general rules for determining suitable involute corrections on spur gears:

a) Along the path of contact, a distance equal to the base pitch should be left void of any correction; and the correction should extend to both sides over more or less equal distances.

b) The correction can be applied to both gears in the form of tip relief, or to one gear alone in the form of tip and root relief. If tip and root relief is applied to both gears, the amounts are simply additive, which means that the individual corrections on each gear are half the total amount. This method is already practised in certain cases, and often brings advantages from the point of view of manufacture.

c) The degree of correction will depend on the specific tooth loading w_g and the accuracy of the gears. For perfectly accurate gears the minimum degree would theoretically be equal to δ_s as per equation 1.

5. Recommendations for the Practical Application of Profile Correction on Spur and Helical Gears

Profile and longitudinal corrections are generally only applied to one gear of a pair; to the pinion, that is, in the form of tip and root relief – and possible crowning. We differentiate between the profile corrections for driving and driven flanks. Along the path of contact, a distance equal to one transverse base pitch will almost invariably be left without correction. Similarly a certain stretch of the facewidth will be left free from longitudinal correction. From the manufacturing aspect, this practice has the important advantage of always leaving a chance to measure directly the two most important dimensions: the base pitch and the helix angle. From the operational aspect, it ensures that the tooth contact conditions of spur gears will still be kinematically correct also under light loads, since the transverse contact ratio is at least equal to 1. Helical gears with adequate overlap ratio are somewhat less sensitive in this respect, as correct kinematic transmission is guaranteed by the effect of the helix. In cases where the specific tooth load is relatively high in relation to the tooth size this rule may be relaxed, and the true-involute portion along the path of contact made shorter. In extreme cases, as for example in aircraft gears, the profile corrections may even extend along the entire tooth flank in order to ensure a smooth blending in of the corrections.[3]

To make things easier for acceptance tests after manufacture, it is advisable to specify tolerance limits for the correction. The tolerance field will be positioned such that a deviation can only have a lessening effect on engagement shocks. For driving and driven gears, then, the fields will lie in opposite directions, in support of the previously described principle of decreasing the driven gear base pitch relative to that of the driving gear.

Taking the general case of a corrected pinion, of the accuracy grade customary in ground gears, typical profile diagrams, as recorded on the involute tester, are shown in Fig. 7 and Fig. 8. These diagrams refer to cases where the thermal influence is not large enough as to require additional corrections.

To attain a smoother profile form by lengthening the the root relief slightly, the tip relief would have to be shortened. This measure should only be resorted to if root corrections are exceptionally short, as is the case with fine-pitched gears.

Degree of Correction for Spur Gears (Equation 3)

w = peripheral unit load in lbs per one inch of facewidth

Δ = corrections in tenthousandths of an inch

At first point of tooth contact:

lower tolerance limit $\Delta_{1u} = 3 + 3.5\,w \times 10^{-3}$

upper tolerance limit $\Delta_{1o} = 6 + 3.5\,w \times 10^{-3}$

At last point of tooth contact:

lower tolerance limit $\Delta_{2u} = 0 + 3.5\,w \times 10^{-3}$

upper tolerance limit $\Delta_{2o} = 3 + 3.5\,w \times 10^{-3}$

Degree of Correction for Helical Gears (Equation 4)

At first point of tooth contact:

lower tolerance limit $\Delta_{1u} = 2 + 2.8\,w \times 10^{-3}$

upper tolerance limit $\Delta_{1o} = 5 + 2.8\,w \times 10^{-3}$

At last point of tooth contact:

lower tolerance limit $\Delta_{2u} = 0 + 2.8\,w \times 10^{-3}$

upper tolerance limit $\Delta_{2o} = 3 + 2.8\,w \times 10^{-3}$

It is known that with high power high speed gears the pinion will gain a higher average temperature than the gear. This results in a difference in base pitch:

$$\Delta p_b = p_b \cdot \Delta\theta \cdot \alpha \qquad \text{(Equation 5)}$$

$\Delta\theta$ Temperature difference pinion/gear
α Thermal expansion coefficient

Corresponding corrections are made by changing the inclination of the tolerance zone of the uncorrected involute section BC, i.e. by correcting the base pitch of the pinion.

In a speed reduction gear a pinion temperature higher than at the gear produces a larger base pitch in the driving member. As described in section 3 and illustrated by Fig. 4 this effect helps, up to a certain point, in reducing tooth engagement shock.

If the temperature difference is of significance, the base pitch difference must be reduced to an acceptable value. This corrective measure imposed on the pinion profile diagram has the effect of lifting point C in Fig. 7.

In a speed increasing gear the situation is reversed. It is the driven member (pinion) attaining a larger temperature and base pitch. This effect, like the tooth deflection, tends to increase the tooth engagement shock (Fig. 3). In order to compensate this temperature influence, the tolerance zone BC on Fig. 8 is given a different inclination again by lifting point C, which is equivalent to reducing the pinion base pitch. An example of such a case is illustrated by Fig. 14b.

For such cases where the average pinion temperature is larger than the gear temperature, the following should be noted:

In a reduction gear the effects of tooth deflection and temperature difference tend to compensate each other. But with a speed increasing gear these two effects are additive. This means that the resultant base pitch correction in a speed increasing gear is larger than in a reduction gear.

6. Factors Influencing the Load Distribution Across the Face of a Gear

Starting from an accurate gear which shows absolutely even load marking across the face width in the unloaded and cold condition we find that the load distribution will not be uniform under service conditions. There are a number of factors responsible for this, which must be kept in mind when designing tooth corrections:

Every pinion under load suffers a certain amount of elastic deformation. The cylindrically shaped pinion body bends and twists under the tooth load. Shear deflections are also present, but they are small and can be neglected.

In a gear which operates at high peripheral speed, it is necessary to check on possible deflections due to centrifugal forces. Depending on the shape and design of the gear body, the toothed area might acquire a slight barrel or concave shape.

There is also a thermal influence to be considered. The higher the power transmitted in one mesh the more pronounced it becomes. The gears get heated unevenly. In a spur gear the temperature is highest in the center of the toothing and drops towards the tooth ends provided the influence of the heat generated in the bearings is negligible. With helical gears the hottest part of the toothing is moved somewhat because of the lube oil being transported axially to one side.

As mentioned above the average pinion temperature is slightly higher than the gear temperature. With helical gears this has the following effect:

Because there are several pairs of teeth in contact simultaneously the temperature difference causing a difference in base pitch (equation 5) has the effect of producing unequal load sharing between the teeth in mesh. In a speed reducer it is the leading tooth pair which takes the highest load (see Fig. 6). The next following tooth pair gets a slightly reduced load, and at every following pair the load decreases. This results in heavier contact marks at one end of the teeth, and gives the impression that the helix angle of the pinion decreases with rising temperature. In actual fact the helix angle remains unchanged. Such one-sided loading is only caused by the base pitch differences, and it obviously becomes more pronounced, the more teeth there are in mesh simultaneously; or in other words, the larger the helix angle.

With single helical gears where the helix angles vary between 6° and 15° this effect is normally very slight. However, it is an advantage to choose the hand of the helix such, that this effect tends to reduce the effect of the pinion torsional deflection due to the tooth load. Fig. 6 shows that in a reduction gear the leading tooth ends of the pinion should be at the pinion coupling side. In a speed increasing gear the situation is reversed; the trailing tooth ends should be at the coupling side.

Other factors influencing the load distribution, like stiffness of the housing and foundation, bearing clearances, etc., must be weighed up for each individual case. It is often an advantage and common practice to make an allowance for such influences when designing the longitudinal corrections.

7. Recommendations for the Practical Application of Longitudinal Corrections on Spur and Helical Gears

In the majority of all cases the main factor which calls for longitudinal tooth corrections are the elastic pinion deflections. The determination of the basic longitudinal corrections is therefore based on these. These deflections can be calculated exactly for a definite transmitted power for which uniform load is desired.

Any other influences on load distribution as mentioned above in section 6 are more difficult to anticipate. For this reason the designer will generally try, in the initial stages of his work, to arrange for these indefinite factors to cancel each other as well as possible, i.e. that they are not additive throughout. The expected misalignment due to these factors is given consideration by superimposing respective corrections on those determined from tooth load deflections. A simple way of calculating elastic pinion deflections and the corrections necessary to ensure optimum load distribution is as follows:

The pinion deflections are determined in a plane tangential to a cylinder of pitch circle diameter (Fig. 9). The tooth load W, which is also acting in this plane, is assumed to be uniformly distributed across the face width. Its value corresponds to the operating load for which optimum load distribution is desired.

The total pinion deflection is composed of two parts, bending (curve 1) and torsion (curve 2). Both act in the same tangential plane. Therefore, the combined deflection (curve 3) is obtained by algebraic addition of the two curves 1 and 2.

In order to compensate elastic deflections under the predetermined load W, the longitudinal correction must be of the shape of the dotted line 4 which is an exact inversion of the combined deflection 3.

For a symmetrically mounted pinion as shown in Fig. 9, the equation for the calculation of the maximum *bending deflection* within the toothed section of face width F is:

$$\delta_b = \frac{2}{E\pi} \cdot w \cdot K^4 \left(\eta - \frac{7}{12}\right) \qquad \text{(Equation 6)}$$

w specific load lbs/in

K face width diameter ratio $\frac{F}{d}$

η bearing span – face width ratio $\frac{L}{F}$

The deflection curve is approximately of circular shape. The maximum value δ_b appears in the middle of the toothing. The influence of the pressure angle is small and can be neglected.

The maximum *torsional deflection* of the toothed section, again assuming uniform load distribution, is:

$$\delta_t = \frac{4}{G\pi} \cdot w \cdot K^2 \qquad \text{(Equation 7)}$$

The torsional deflection curve is of parabolic shape; its vertex being at the tooth ends away from the coupling.

If the toothed part of the pinion has a bore of diamete d_i the above values δ_b and δ_t must be multiplied by:

$$K_i = \frac{1}{1 - \left(\frac{d_i}{d}\right)^4} \qquad \text{(Equation 8)}$$

For a quick determination of the combined pinion deflection δ the curves of Fig. 10 can be applied. They are plotted as a function ot the face width diameter ratio K and are based on the following data:

Curve A:

Pinion in mesh with *one* gear
Pinion symmetrically mounted as shown in Fig. 9
Bearing span – face width ratio $\eta = 1.7$
Unit load $w_o = 100$ lbs/in

The maximum combined deflection δ for any load w is then:

$$\delta = \delta_{100} \cdot \frac{w}{100} \quad \text{in tenthousandths} \qquad \text{(Equation 9)}$$

Curve B:

Pinion in mesh with *two* gears, 180° displaced as shown in Fig. 9.
Unit load per mesh $w_o = 100$ lbs/in
The maximum combined deflection δ is again obtained from equation 9.
It must be noted that w is the unit load for *one* mesh only.
If the pinion engages three gears as for instance in an epicyclic gear, the values δ_{100} given by curve B are simply multiplied by $\frac{3}{2}$.

Manufacturing and inspection techniques make it desirable to deviate somewhat from the theoretically determined form of correction according to curve 4 in Fig. 9. Practical experience supplies the necessary directives. As with profile corrections, part of the tooth is left uncorrected. This uncorrected portion assures adequate overlap ratio ($\geqq 1$ if possible) and hence smooth running when operating under light loads. It also permits a direct measurement of the helix angle. In such cases where loading and deformations are extreme, the above principle can no longer be applied, for the correction must extend across the full face width.

For combined pinion deflection δ not exceeding ~[illegible] tenthousandths, it has become sound established practice to give the longitudinal correction the form as shown in Fig. 11 and Fig. 12.

The degree of correction is calculated from the combined deflection δ, and provided with suitable tolerance limits for manufacture and inspection.

Degree of Longitudinal Correction for Single Mesh (Equation 10)

Form of correction see Fig. 11.

Coupling end tenthousandths

Lower tolerance limit $\Delta_{xu} = \delta$
Upper tolerance limit $\Delta_{xo} = \delta + 2$

Blind end

For $K \leqq 1$ --------------- $\Delta_y = 0$

For $K > 1$
Lower tolerance limit $\Delta_{yu} = 0.3\ \delta$
Upper tolerance limit $\Delta_{yo} = 0.3\ \delta + 2$

Degree of Longitudinal Correction for Double Mesh (Equation 11)

Form of correction see Fig. 12.

Coupling end

Lower tolerance limit $\Delta_{xu} = \delta$
Upper tolerance limit $\Delta_{xo} = \delta + 2$

Blind end $\Delta_y = 0$

8. Field Experience

Tooth corrections have been successfully applied since pinion grinding machines equipped with correction facilities became available about 15 years ago.

These machines are capable of producing accurately controlled corrections within one tenthousandth of an inch.

Since then, this technique of finely controlled correction has been applied to a vast number of hardened and ground gears of various sizes and all kinds of application.

Observations on these gears in service have supplied adequate proof of the soundness of the principles of correction as described in this paper.

The following examples are presented as typical concrete cases of successfully applied tooth correction.

Example 1: Fig. 13

Marine Reduction Gear between Diesel Engine and Screw Shaft

Max. power	3600 HP
Speeds	1640/739 rpm
Pinion diameter	d = 8.3"
Face width	F = 10.2"
Spec. tooth load	w = 3200 lbs/in
k-factor	560
Helix angle	$\psi = 10°$
Diametral pitch	$P_d = 3.85$

Gears carburized, hardened and ground

Profile Corrections:

The degree of correction is evaluated from equation 4.

$$\Delta_{1u} = 2 + 2.8 \times 10^{-3} \cdot w = 11 \text{ tenthousandths}$$
$$\Delta_{1o} = 5 + 2.8 \times 10^{-3} \cdot w = 14 \text{ tenthousandths}$$
$$\Delta_{2u} = 0 + 2.8 \times 10^{-3} \cdot w = 9 \text{ tenthousandths}$$
$$\Delta_{2o} = 3 + 2.8 \times 10^{-3} \cdot w = 12 \text{ tenthousandths}$$

The profile diagram of the pinion teeth is shown on Fig. 13b. It is based on the recommendations given by Fig. 7. The tolerance zone of the profile section between points B and C allows a deviation from the true involute of 1.2 tenthousandths. This is equivalent to a pinion base pitch being up to 1.4 tenthousandths larger, but never smaller, than the theoretical value.

Longitudinal Corrections

Face width diameter ratio $K = \frac{10.2}{8.3} = 1.23$

From Fig. 10, curve A : $\delta_{100} = 0.2$ tenthousandths

Combined deflections (equation 9)

$$\delta = \frac{w}{100} \cdot \delta_{100} = \frac{3200}{100} \cdot 0.2 = 6.4 \text{ tenthousandths}$$

From equation 10 one obtains the degree and from Fig. 11 the form of the correction.

Coupling side: $\Delta_{xu} = \delta = 6.4$ tenthousandths
$\Delta_{xo} = 2 + \delta = 8.4$ tenthousandths
Blind side: $\Delta_{yu} = 0.3\ \delta = 2$ tenthousandths
$\Delta_{yo} = 2 + 0.3\ \delta = 4$ tenthousandths

The longitudinal diagram is plotted in Fig. 13c.

Example 2: Fig. 14

Speed Increasing Gear between Twin Diesel Engine and Generator for Diesel Electric Locomotive

Two Diesel crank shafts drive one generator. The layout of the gears is shown in Fig. 14a.

There are rigid couplings on the input and output shafts.

Power per crankshaft		:	1175 HP
Speed of crankshafts		:	750 rpm
Speed of generator		:	1078 rpm
Pinion diameter	d	=	13.55"
Face width	F	=	11.42"
Diametral pitch	P_d	=	2.36
Helix angle	ψ	=	0°
Backlash		=	$3\frac{1}{2} \div 4\frac{1}{2}$ thousandths
Average tooth load w_o		=	873 lbs/in
Torque Variation at normal load		=	±300%

Gears carburized, hardened and ground

This is an unusual case because of the high torque fluctuation at normal operation, which accounts for the small backlash.

The peak load on the teeth is four times the average load:

$$w_{max} = \sim 3500 \text{ lbs/in}$$

Observations made on this gear in service confirmed that not only elastic deflections, but also thermal influences must be considered when determining the flank correction required. The heat developed in the bearings causes the diameter to grow more at the tooth ends than in the middle. Gears of this type which were made without longitudinal correction suffered scoring at the tooth ends. To rectify the matter, the symmetrical longitudinal correction shown in Fig. 14b was applied, and no further trouble was experienced.

Whereas no actual correction is applied to the teeth of the mating wheels, the tolerance zone for the tooth alignment is positioned such as to compensate for the very slight torsional deflections.

The degree of tip and root relief shown in Fig. 14b corresponds to values obtained by substituting into equation 3 a tooth load w equal to approximately double the average load w_o.

Since the pinion is driven by two wheels, a certain temperature difference between wheels and pinion must be expected. In a speed increasing gear like the one in question, the temperature difference and the tooth deflections have an additive effect as mentioned earlier. The rule here is to make the base pitch of the pinion 1.5 to 2.5 tenthousandths of an inch below the theoretical value.

In all cases where thermal influences are felt, and where appreciable vibrational forces or shocks occur, practical experience is the only reasonably reliable guide. But the delicate control possible in correction grinding permits the faithful reproduction of even the finest corrections which such field experience ... suggest as being necessary.

Example 3: Fig. 15 and 16

Pinions for Rolling Mill

Pinion diameter	d	= 14"
Center distance	a	= 14"
Effective face width	F	= 2 × $8\frac{1}{4}$" (double helical)
Gap		= $4\frac{1}{2}$"
Total face width	F_t	= 21" (2F + gap)
Diametral pitch	P_d	= 1.429
Helix angle	ψ	= 28°

Teeth carburized, hardened and ground

Input torque acting on one pinion:

Normal torque	$T = 0.97 \cdot 10^6$ lbs/in
Maximum torque	$T_{max} = 1.40 \cdot 10^6$ lbs/in

The output torque is equally divided between the two pinions. Only half the input torque is transmitted by the teeth. The other half is transmitted directly by the input pinion to the output coupling.

$$\text{Specific load } w = 4200 \text{ lbs/in } (k = 600)$$
$$w_{max} = 6100 \text{ lbs/in } (k_{max} = 870)$$

Because this rolling mill operates only part time at the max. torque it was decided to apply corrections based on the normal load w.

Profile Corrections:

Because the pinions are of the same dimensions and the necessary tip and root relief is rather large, the corrections are shared equally between both. However, the tolerance zones of the true-involute flank portions are opposed to each other, such that the base pitch of the driving pinion can never be smaller than the pitch at the driven pinion, see Fig. 15.

The degree of the tip and root corrections were derived from equation 4.

Longitudinal Corrections:

In a case like this the combined deflections δ can no longer be read from graph Fig. 10. The various deflections must be calculated separately and superimposed.

In this case the formulae 6 and 7 give a close approximation for the deflections δ_b and δ_t respectively, if the tooth load is assumed to be evenly distributed across the total face width F_t (see Fig. 16). The imaginary specific tooth load w_o is then:

$$w_o = w \cdot \frac{2F}{F_t} = 4200 \frac{2 \cdot 8\frac{1}{4}}{21} = 3300 \text{ lbs/in}$$

The face width diameter ratio is $K = \frac{F_t}{d} = \frac{21}{14} = 1.5$

Fig. 16 shows the combined deflection curves for the driving and driven pinion:

Driving: Combined deflection curve 4a = 1 + 2 + 3
Driven: Combined deflection curve 4b = 1 + 2

Longitudinal correction is only applied to the driving pinion. The resultant of curves 4a and 4b supplies the total combined corrections at the helix ends. They are (compare Fig. 16):

Coupling side $\delta_x = 15.5 + 2.7 = 18.2$
Blind side $\delta_y = 9 - 4 = 5$

At the bottom of Fig. 16 is shown the longitudinal diagram, as it was executed for this particular case.

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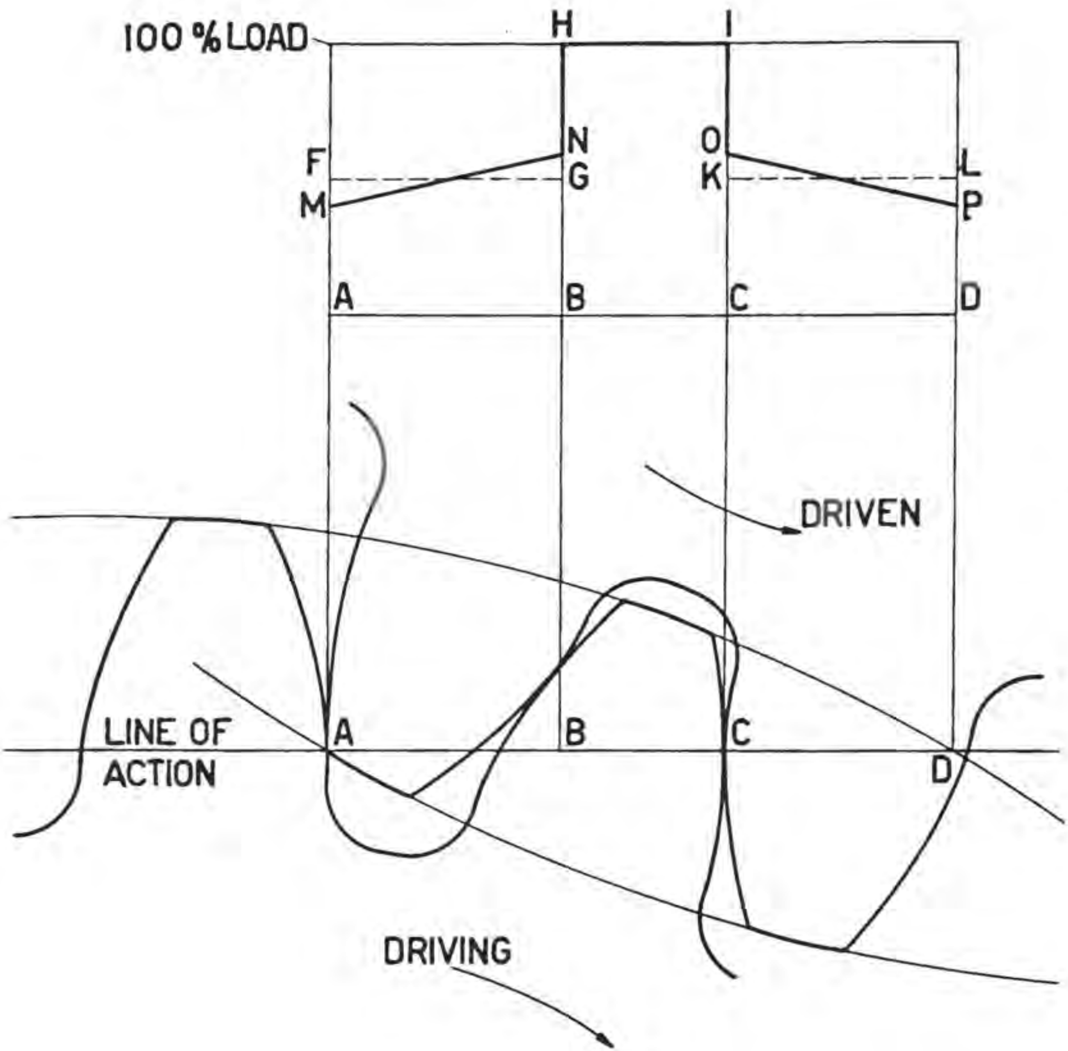


FIG. 2 LOAD DISTRIBUTION ON TRUE-INVOLUTE SPUR GEARS

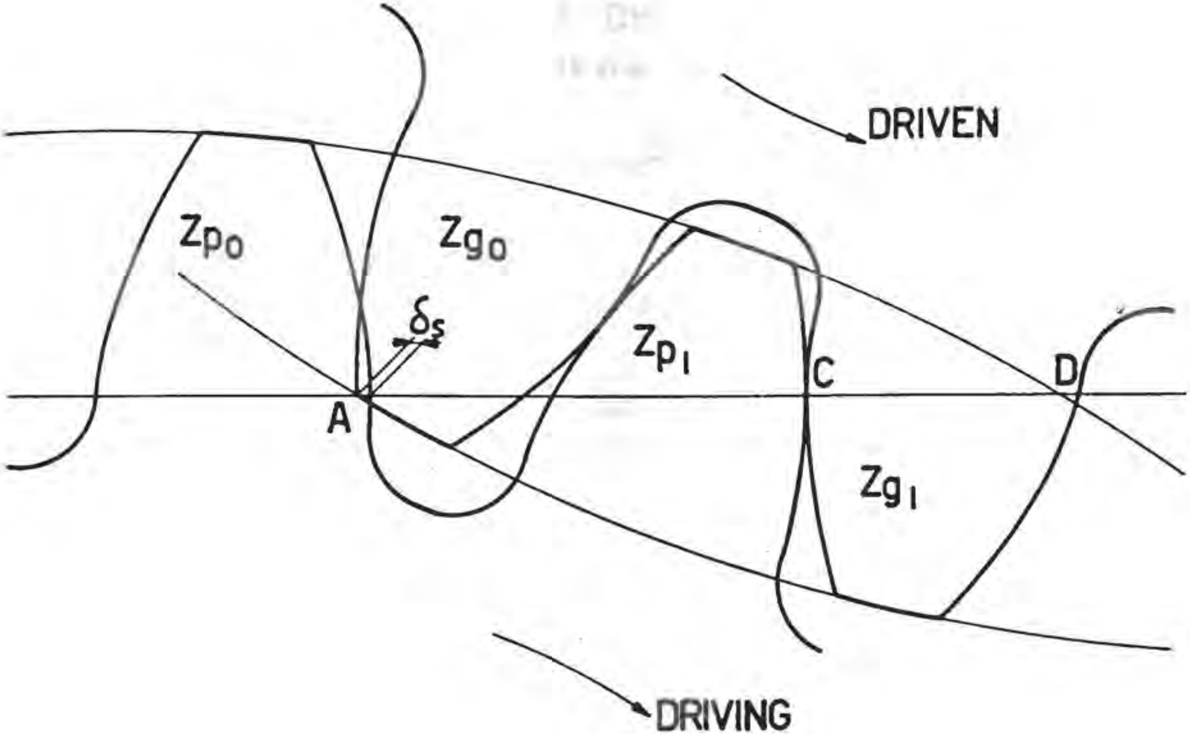


FIG. 3 TOOTH INTERFERENCE ON LOADED TRUE-INVOLUTE GEARS

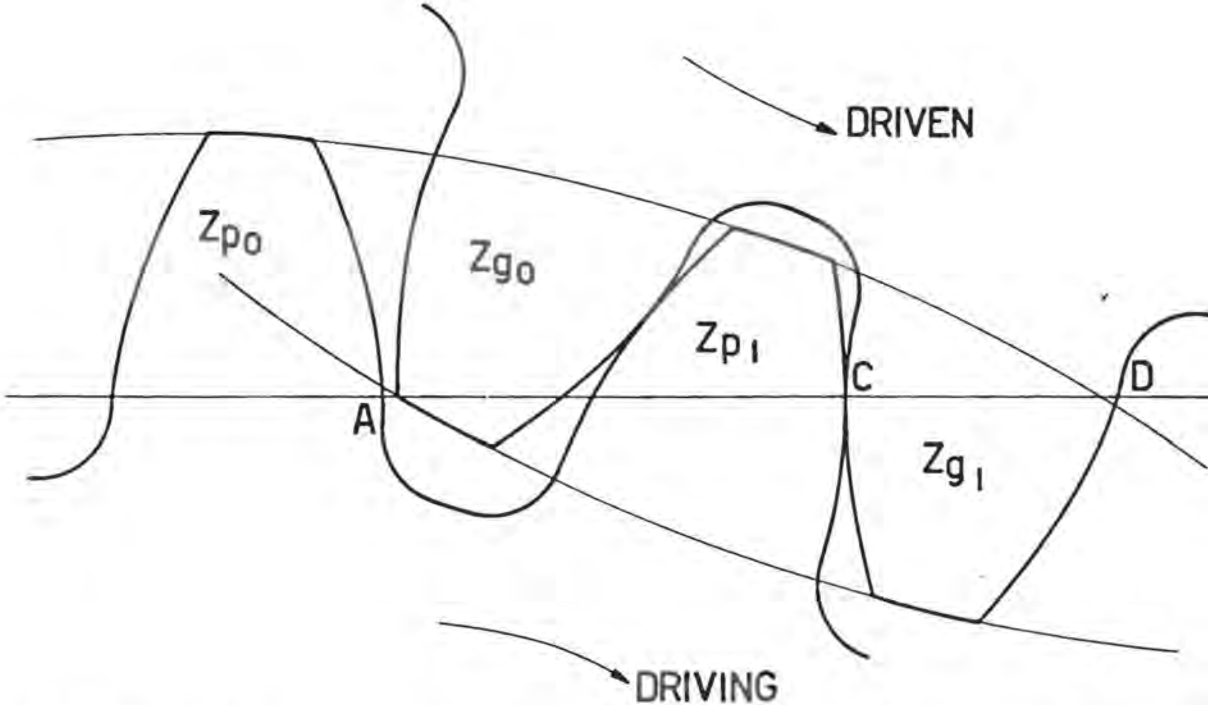


FIG. 4 REDUCING ENGAGEMENT SHOCK BY ENLARGING THE BASE PITCH OF THE DRIVING GEAR

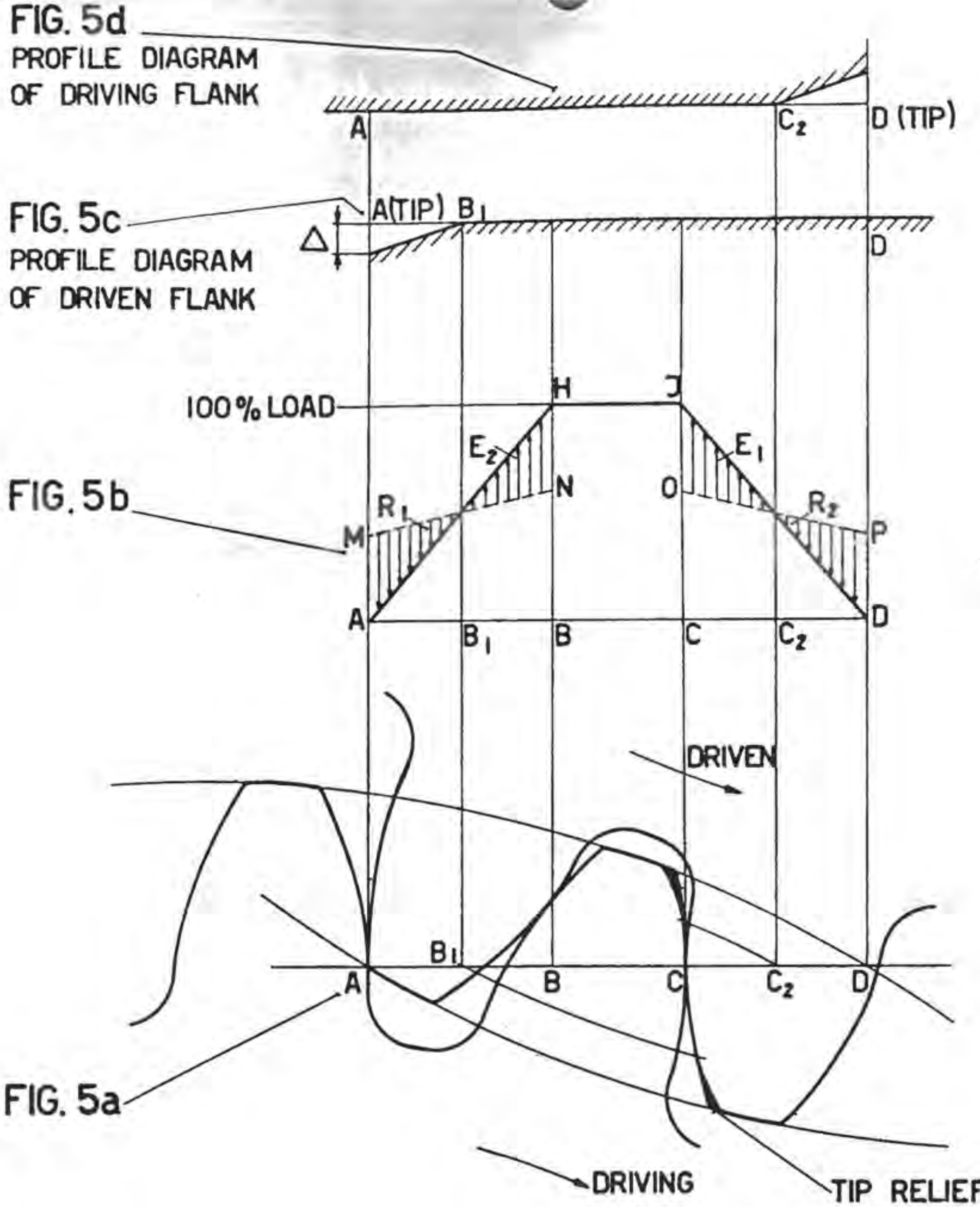


FIG. 5 LOAD DISTRIBUTION AND PROFILE CORRECTIONS

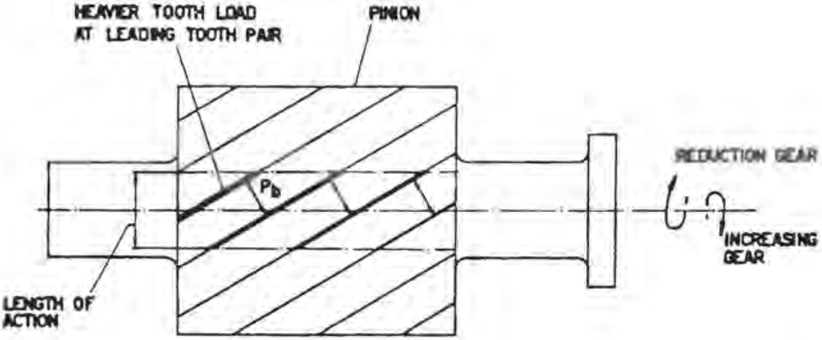
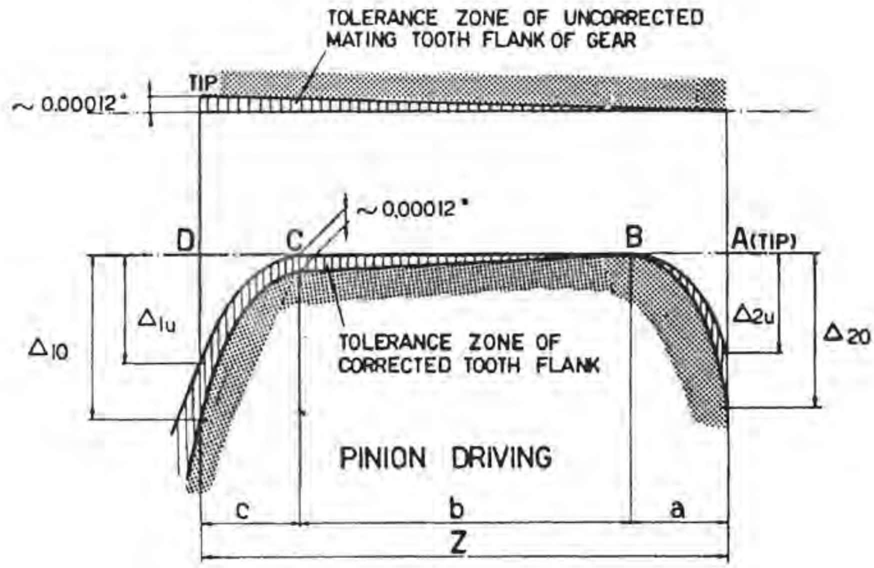


FIG. 6 EFFECT OF TEMPERATURE DIFFERENCE BETWEEN PINION AND GEAR



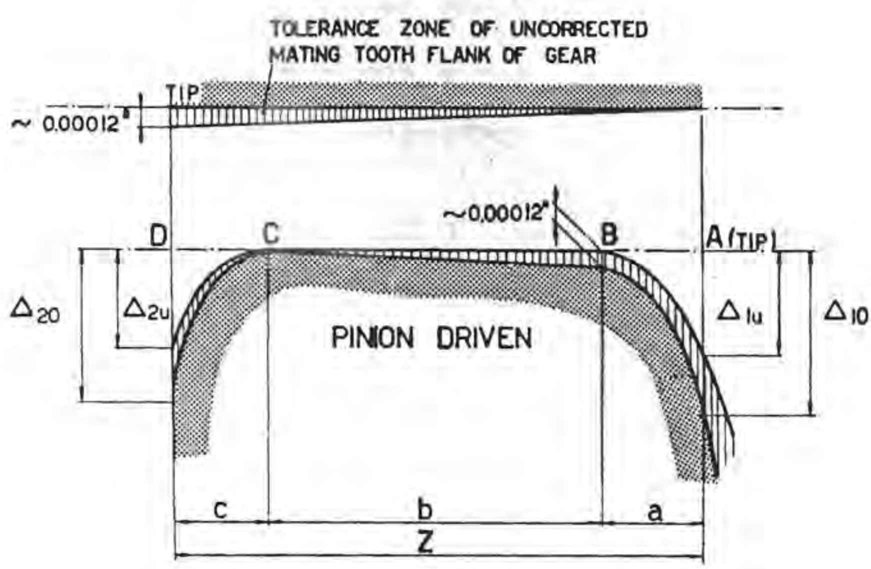
Z = LENGTH OF CONTACT (OBTAINED FROM DIMENSIONS OF THE GEARS IN MESH)

P_b = TRANSVERSE BASE PITCH

$a = \frac{Z - P_b}{2}$ (OR UP TO 20% SMALLER)

$c = \frac{Z - P_b}{2}$ (OR UP TO 20% LARGER)

FIG. 7 PROFILE CORRECTIONS ON A SPEED REDUCTION GEAR



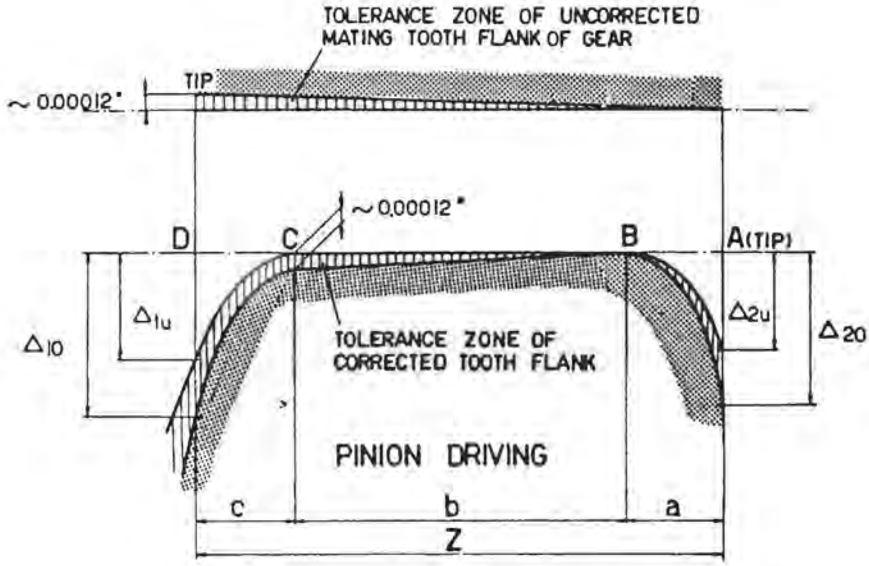
Z = LENGTH OF CONTACT (OBTAINED FROM DIMENSIONS OF THE GEARS IN MESH)

P_b = TRANSVERSE BASE PITCH

$a = \frac{Z - P_b}{2}$ (OR UP TO 20% SMALLER)

$c = \frac{Z - P_b}{2}$ (OR UP TO 20% LARGER)

FIG. 8 PROFILE CORRECTIONS ON A SPEED INCREASING GEAR



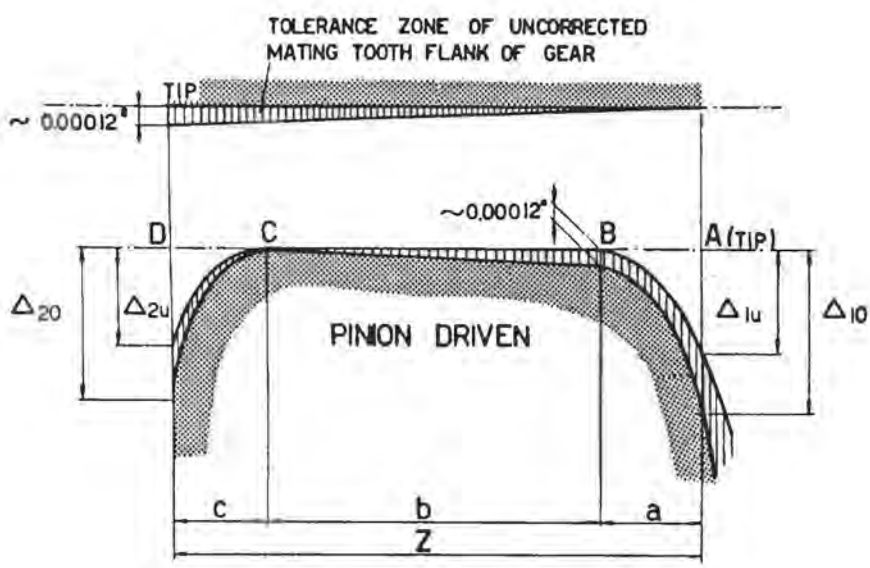
Z = LENGTH OF CONTACT (OBTAINED FROM DIMENSIONS OF THE GEARS IN MESH)

P_b = TRANSVERSE BASE PITCH

$a = \frac{Z - P_b}{2}$ (OR UP TO 20% SMALLER)

$c = \frac{Z - P_b}{2}$ (OR UP TO 20% LARGER)

FIG. 7 PROFILE CORRECTIONS ON A SPEED REDUCTION GEAR



Z = LENGTH OF CONTACT (OBTAINED FROM DIMENSIONS OF THE GEARS IN MESH)

P_b = TRANSVERSE BASE PITCH

$a = \frac{Z - P_b}{2}$ (OR UP TO 20% SMALLER)

$c = \frac{Z - P_b}{2}$ (OR UP TO 20% LARGER)

FIG. 8 PROFILE CORRECTIONS ON A SPEED INCREASING GEAR

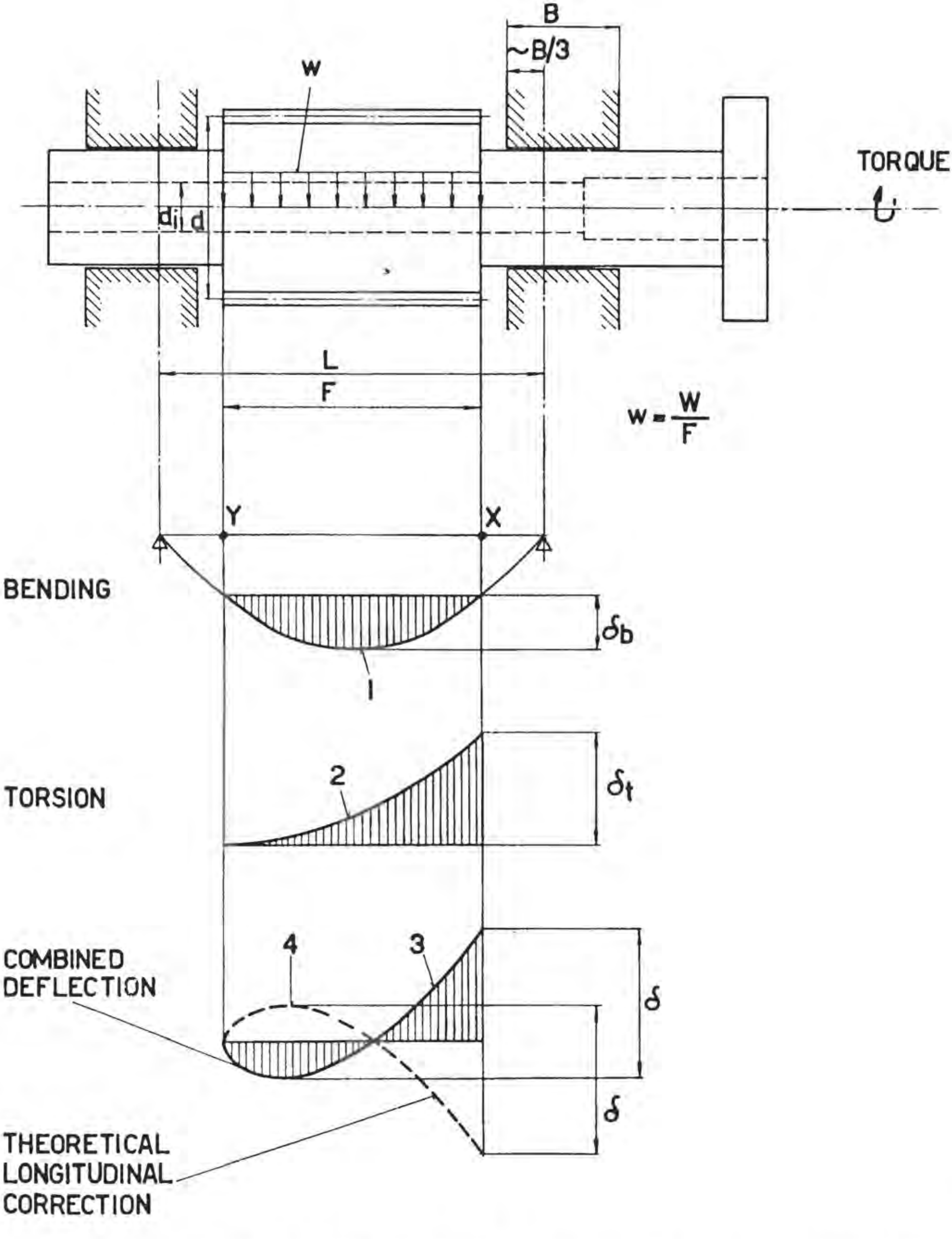


FIG. 9 PINION DEFLECTIONS AND LONGITUDINAL CORRECTIONS

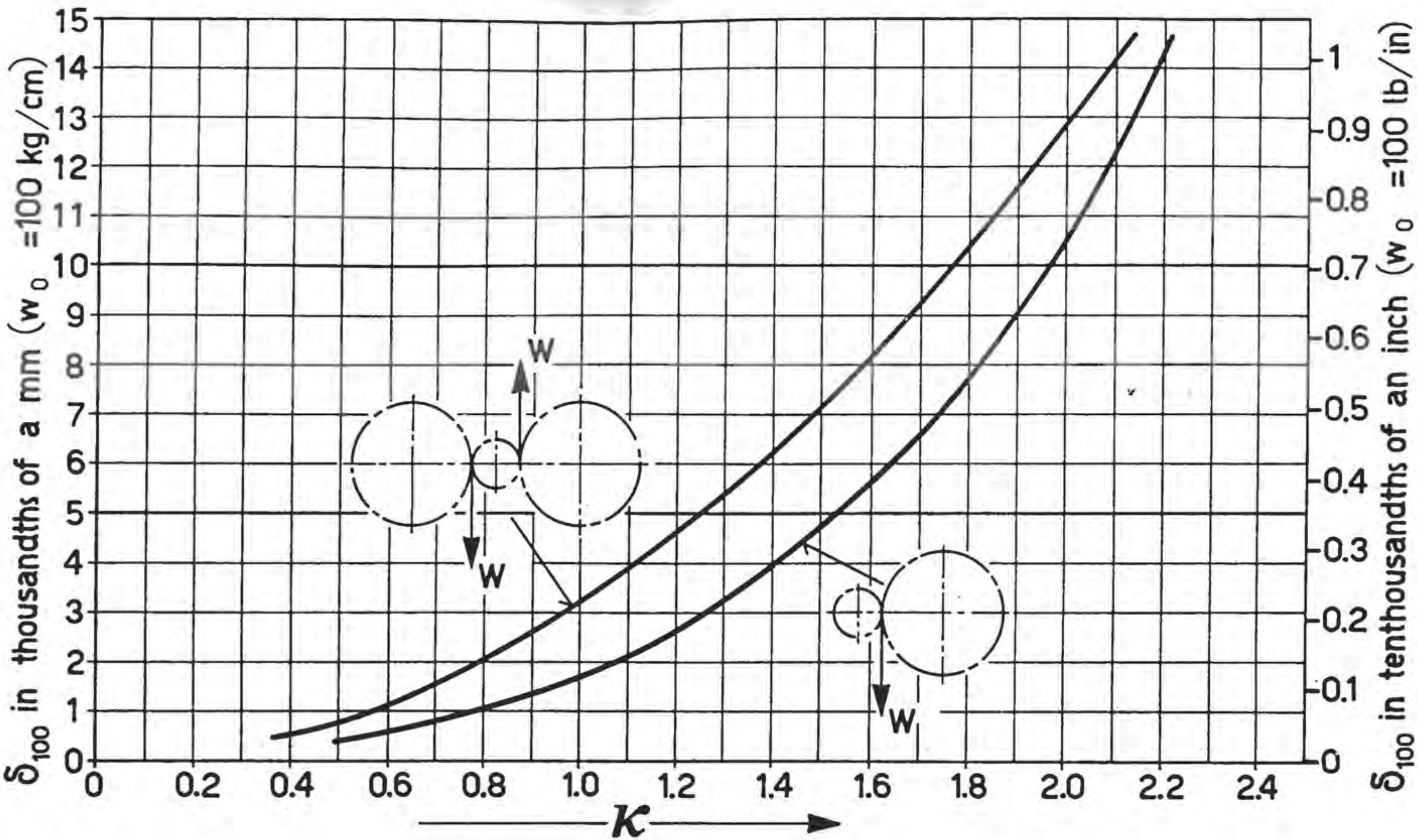
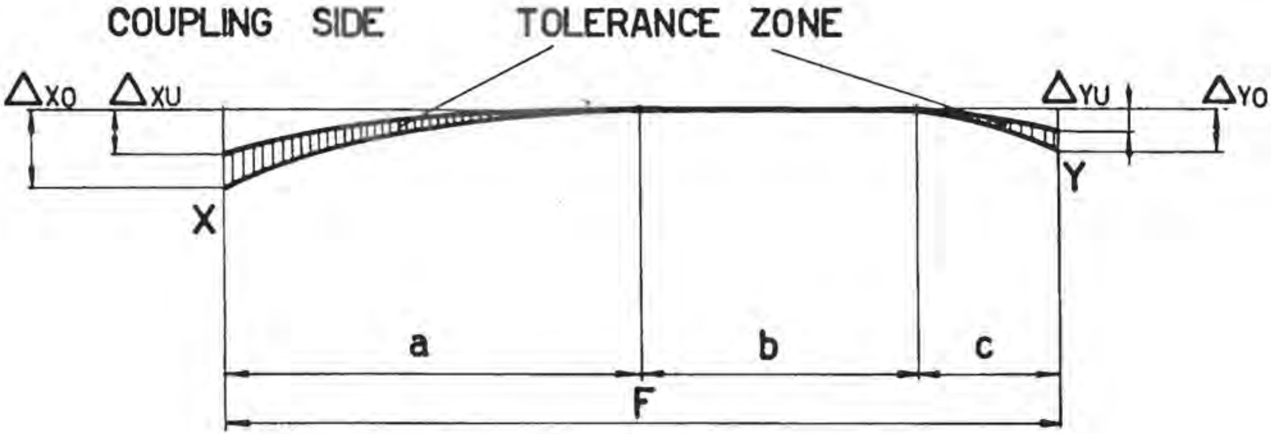


FIG. 10 COMBINED PINION DEFLECTION δ_{100} AT UNIT LOAD $w_0 = 100$ lbs / in



	$K \leqq \sim 1$	$K > \sim 1$
a	$\sim \frac{1}{2} F$	$\sim \frac{1}{2} F$
b	$\sim \frac{1}{2} F$	$\sim \frac{1}{3} F$
c	0	$\sim \frac{1}{6} F$

FIG. 11 LONGITUDINAL PINION CORRECTIONS FOR SINGLE MESH

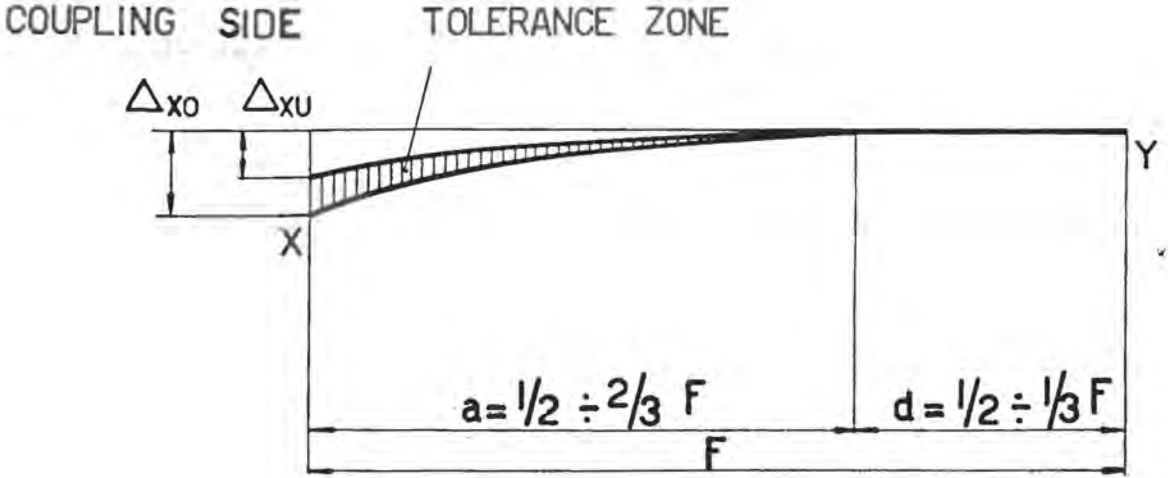


FIG. 12 LONGITUDINAL PINION CORRECTIONS FOR DOUBLE MESH

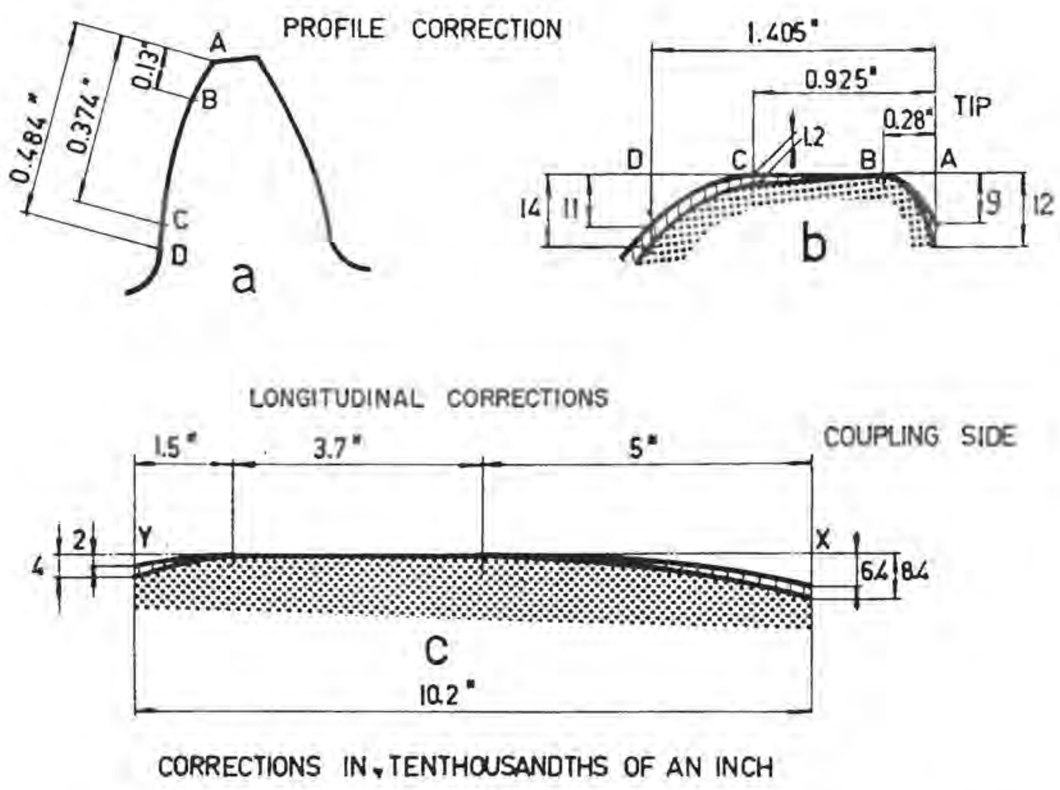


FIG. 13 PINION TOOTH CORRECTIONS FOR EXAMPLE 1

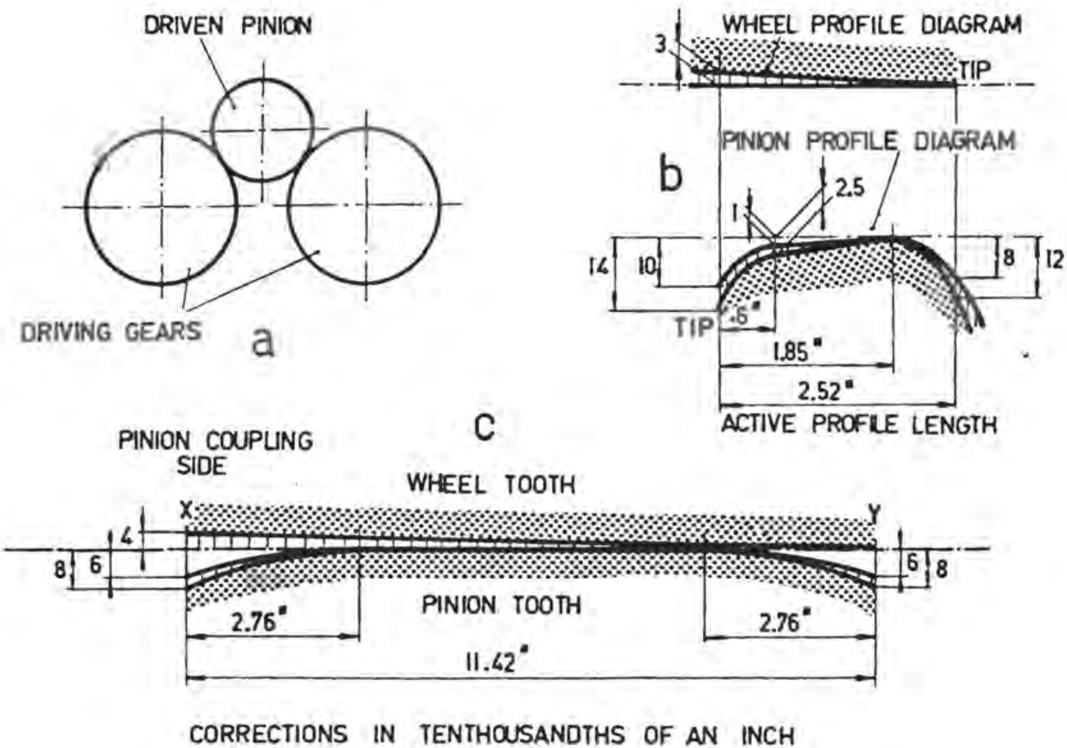
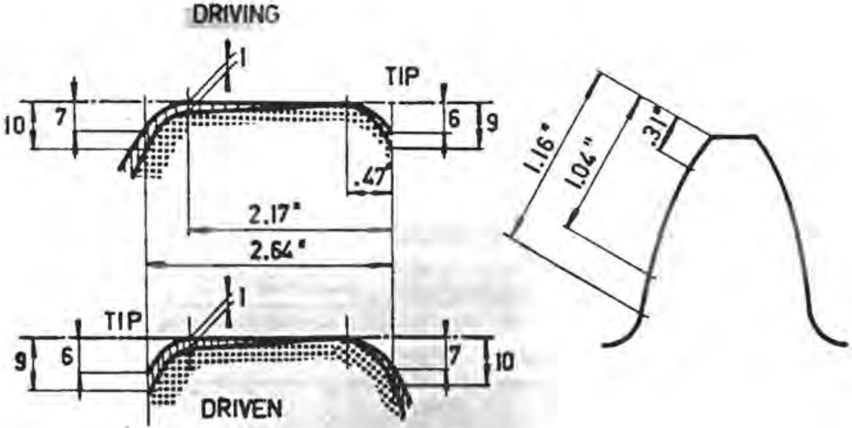


FIG. 14 TOOTH CORRECTIONS FOR EXAMPLE 2



CORRECTIONS IN TENTHOUSANDTHS OF AN INCH

FIG. 15 PROFILE CORRECTIONS FOR EXAMPLE 3

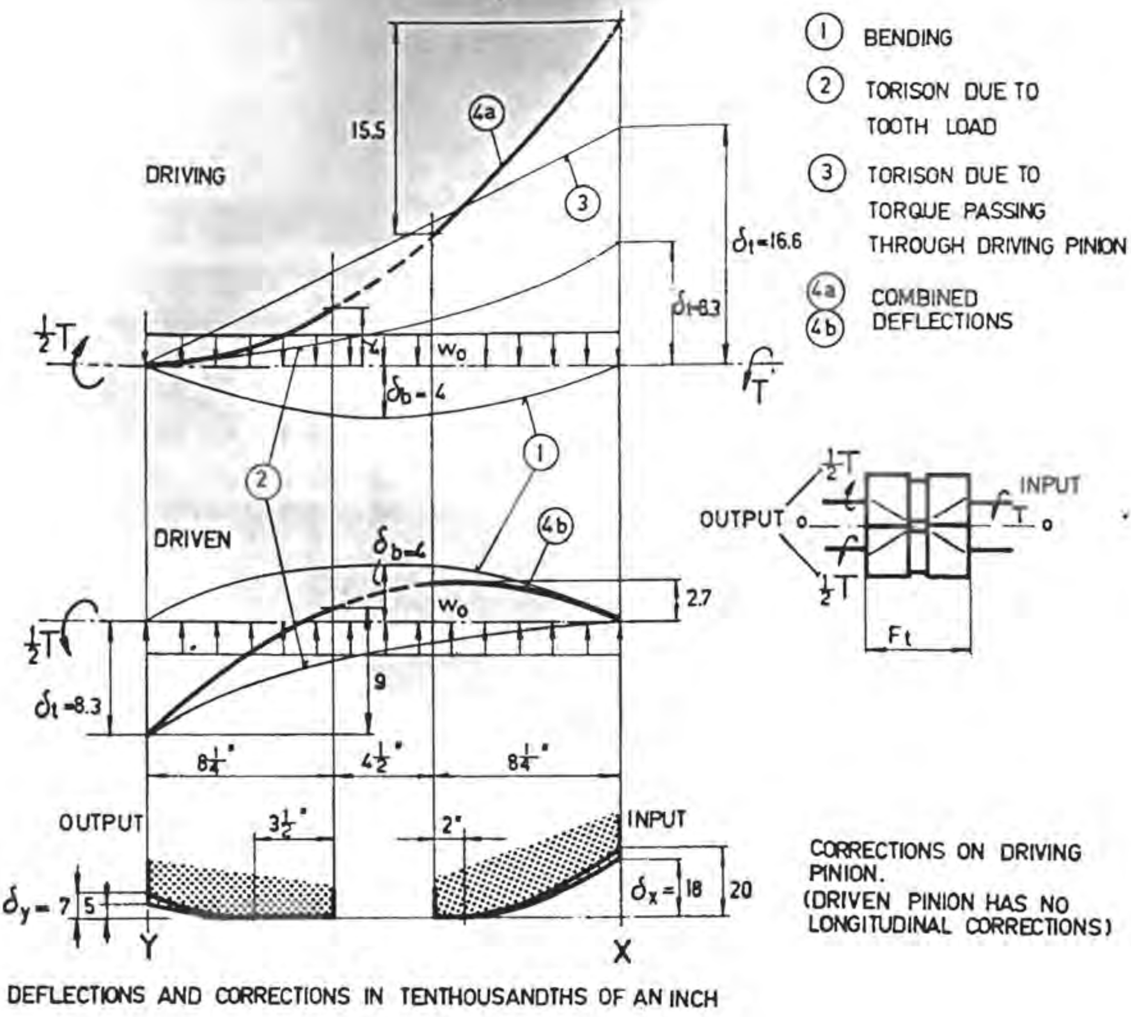


FIG. 16 LONGITUDINAL CORRECTIONS FOR EXAMPLE 3