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Thermal Instability in High-Speed Gearing

Thermal-expansion effects in gearing have usually been considered from a steady-state point of view. A theory of thermal instability is developed which takes into account the tendency of the thermal effects to be regenerative. This theory provides an adequate and complete explanation for several previously unexplained cases of tooth failure in high-speed high-horsepower reduction gears. Experimental verification of the theory is presented and some of the conditions for avoiding thermal instability are described.

THE past 10 or 15 years have been a period of great progress in the manufacture and application of high-horsepower high-speed reduction gearing. This has been particularly true in the field of turbine-driven ship-propulsion gearing, especially for naval vessels, in which field the design departments of the United States Navy and on smaller ships other navies have displayed engineering boldness in applying high-horsepower gears at nearly double the specific loading that prevailed in 1945.

This progress has not been achieved without a few cases of serious reduction-gear trouble, several of these cases having occurred during full-load testing of the reduction gears at the factory and others of which having occurred on ships during trials or later. These difficulties have for the most part been of the type in which a small segment of a tooth breaks out by fatigue, although one or two of the cases have comprised minor profile distress by scuffing. With the harder materials used in these higher-K-factor gears, profile deterioration by surface fatigue, commonly known as pitting, has not proved to be a significant problem, so that the design advances that have made the highly loaded marine gear practicable have thus overcome what was formerly the major factor limiting the service life of the reduction gear, namely, profile failure by pitting.

This paper is concerned mainly with tooth failure by fatigue in bending. Since the time of the first failures of this type on large propulsion gears, strenuous efforts have been made to understand and explain the phenomena [1, 2, 3].[1] The explanations that have been advanced have been partial, at least quantitatively, since the nominal factor of safety in bending fatigue has in every case been of the order of 3. It is believed that the theory and data in this paper will comprise a more adequate explanation of one class of these failures.

Generally, the discussion will pertain to double-helical high-speed gearing, operating at pinion speeds in the 4000 to 8000-rpm range, at pitch-line speeds up to 300 fps, and transmitting up to approximately 15,000 hp per mesh. The specific size of a reduction gear is determined by the K-factor. This number is in the vicinity of 200 for most of the gears under discussion.

Since the power rating of these gears is near the limit of existing skills and manufacturing capability, any serious exploration ot subtle difficulties must be accompanied by full-scale experimental work. Since 1946, the Bureau of Ships has conducted a vigorous program of full-scale testing of ship reduction gears, of both conventional and experimental designs, at the Naval Boiler and Turbine Laboratory [4]. When we encountered difficulty with the main reduction gears for a large aircraft carrier during production testing in our shop several years ago, we immediately commenced to build a full-scale model of the first reduction elements in our Steam Division laboratory at Westinghouse for experimental investigation.

Operation over a long period has produced a strange anomaly—when the gears are operated under conditions seemingly identical to the ship's gears at loads and powers in excess of 200 per cent rating for very long periods of time, there is no distress whatsoever, even when the gearing is subjected to certain intentional derangements.

A repetition during shop testing of prior tooth-breakage difficulties caused an intensified examination of the evidence and a thorough review of the experiments performed. The present theory of thermal instability of gearing was developed. Then it became evident that the experimental testing had not been done in a manner identical to the operating conditions of the ship's gear.

The laboratory test was then altered to reproduce the conditions more completely. At 100 per cent load, indications of distress began to appear, and at 140 per cent load an identical tooth failure was produced in less than 6 hr of operation.

Methods of design and manufacture have been developed for the avoidance of this type of tooth failure.

Favorable Laboratory Results

The laboratory test of high-speed gearing, Fig. 1, consists of two locked-train sets of aircraft carrier first reduction gears, connected in a back-to-back arrangement with a torquing coupling on the shaft between the two high speed pinions. Drive is by a small turbine coupled to one of the gears.

Data on the gearing under test, Table 1, show that the K-factor and tooth bending stress are moderately high and that the pitch-line speed is high.

Whereas the size of the test is very large, the unique feature of the experiment is the use of strain gages, Fig. 2, in the roots of the teeth to measure tooth-load distribution and bending stress [1]. Three gages are arranged across the face width of each helix, Fig. 3. In addition, three resistance-wire temperature gages are placed at the two ends and at the gap of the pinion. The nine gage signals are taken from the rotating shaft through slip rings. The pinion on each unit is instrumented.

The sketch, Fig. 4, shows the arrangement of gears and pinions, the direction of rotation, and the designation of the two gear units as the A unit and the B unit. The results of a few of the many very favorable tests on this apparatus are given in the following paragraphs.

Tests at Increasing Loads. Since the pinion-material endurance limit is 70,000 psi minimum, as determined by rotating-beam specimens cut from similar pinion forgings, the apparent factor of safety for bending fatigue is approximately 4. One would therefore expect that this gearing would take large overloads without tooth fracture, barring any severe irregularities in tooth loading due to poor alignment, incorrect helical angle, or tooth non-straightness.

[1] Numbers in brackets designate References at end of paper.

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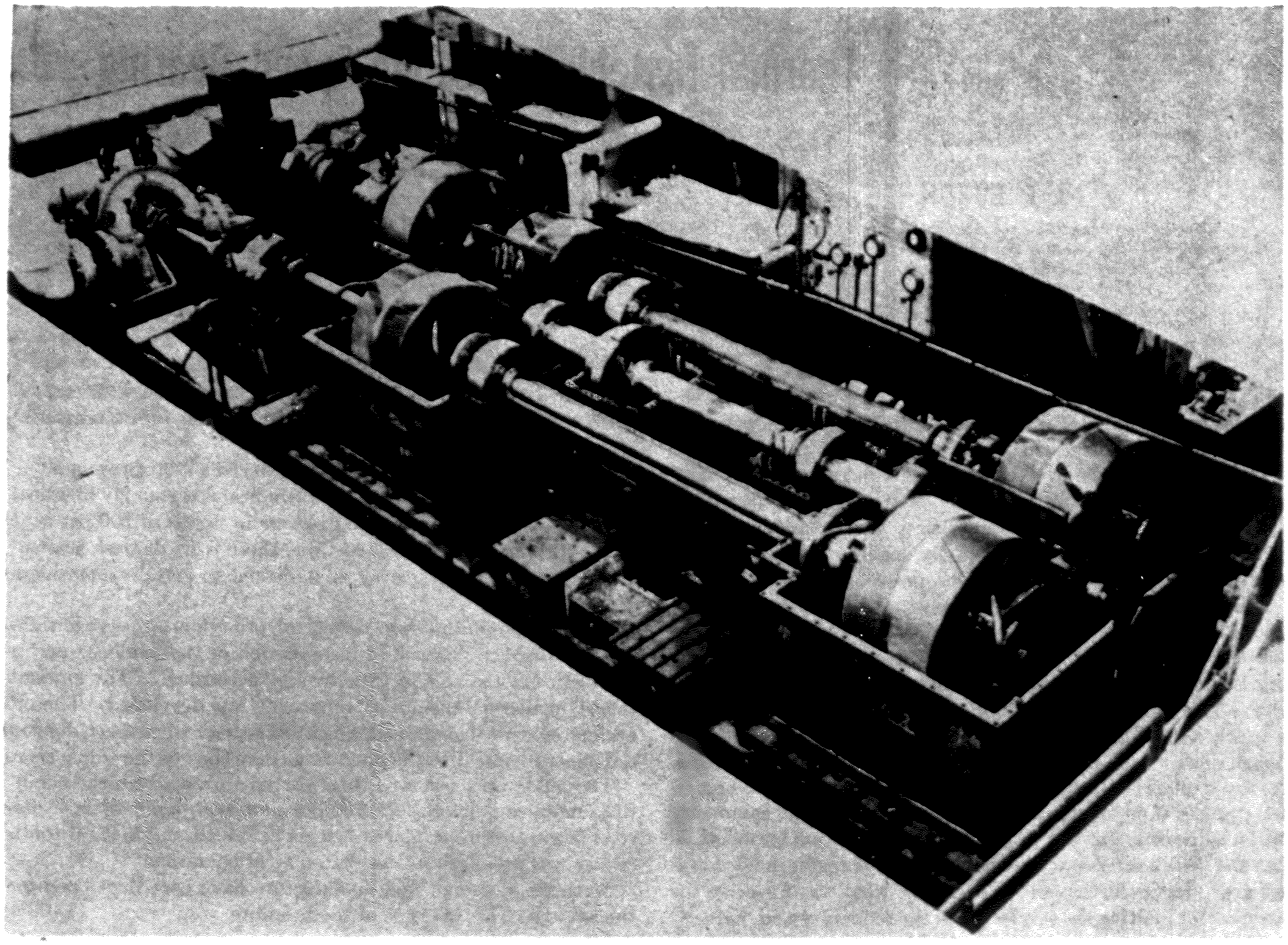


Fig. 1 Laboratory test of high speed gearing

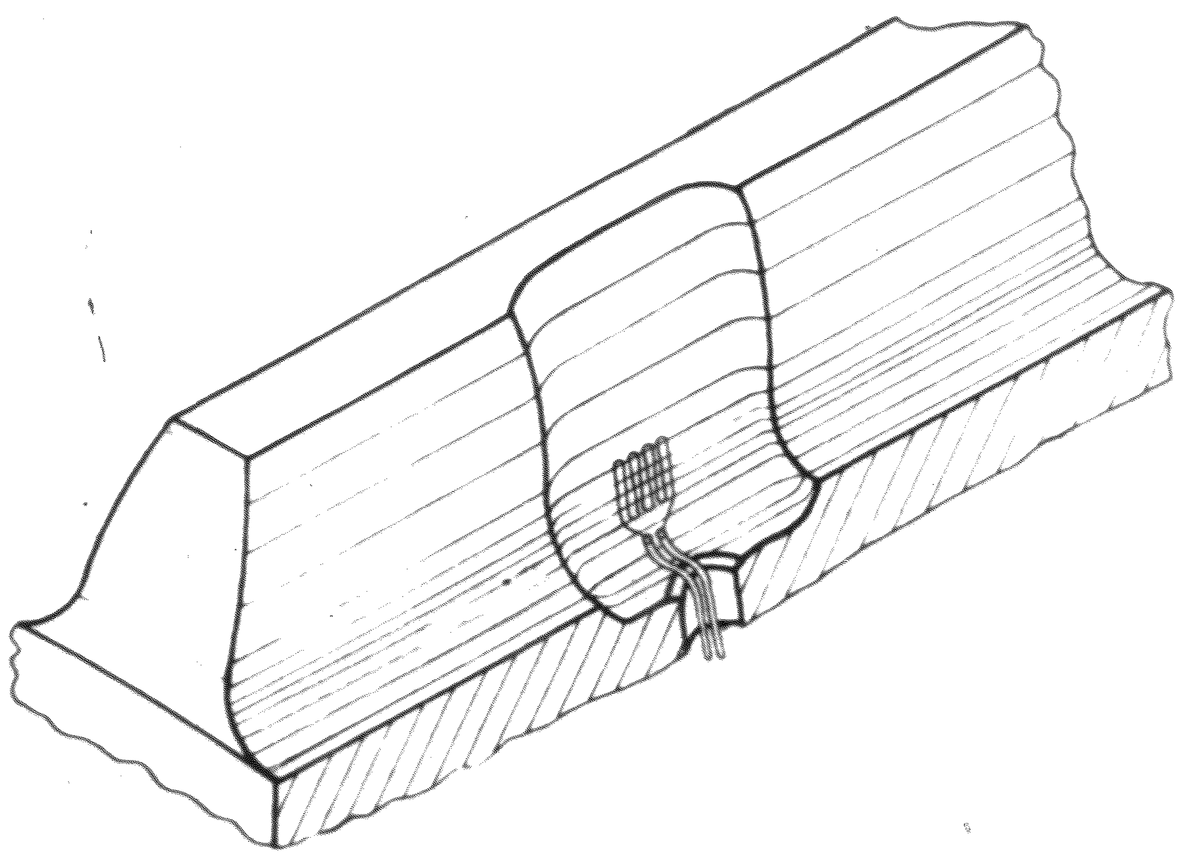


Fig. 2 Strain gage in tooth root

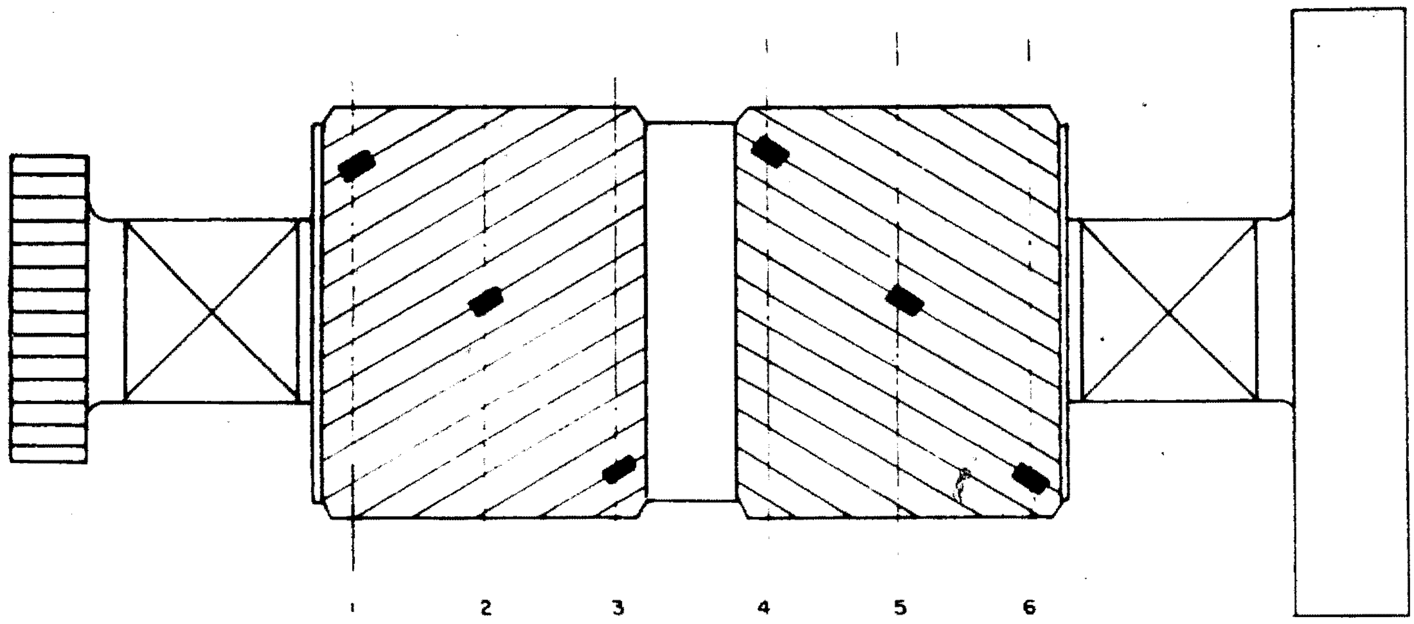


Fig. 3 Location of gages on pinion

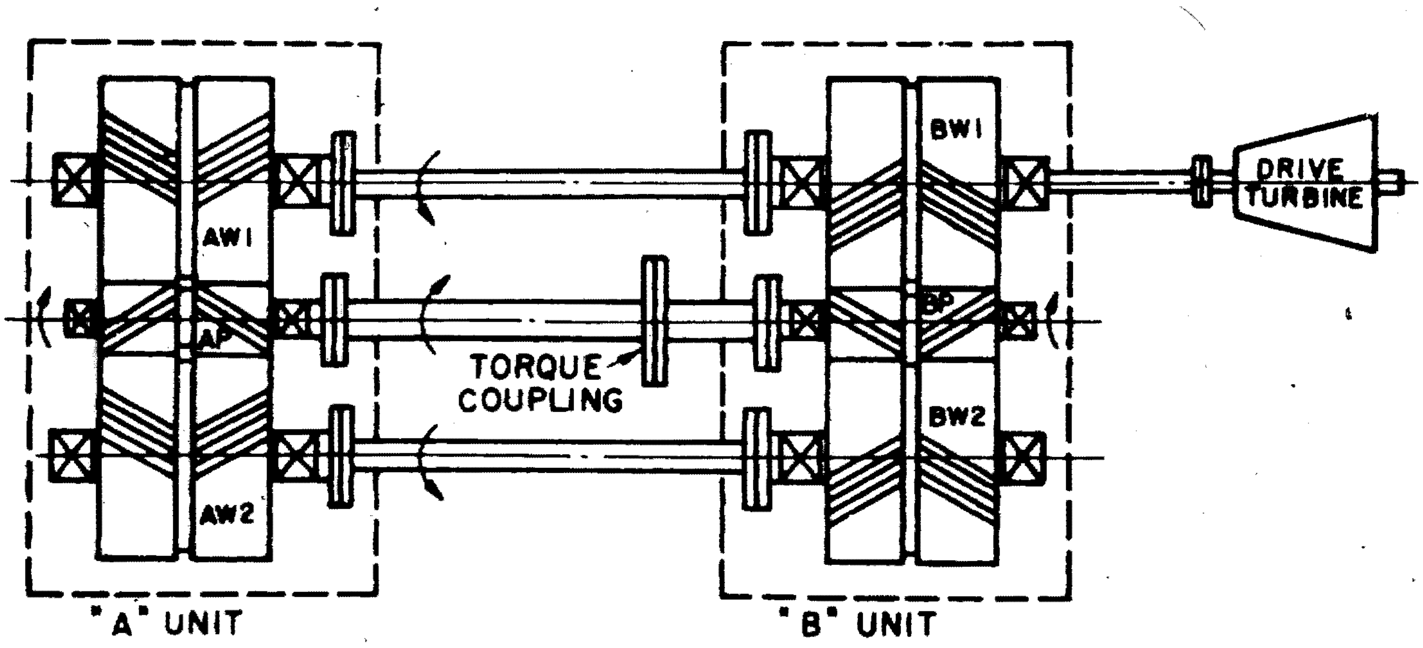


Fig. 4 Sketch of laboratory test

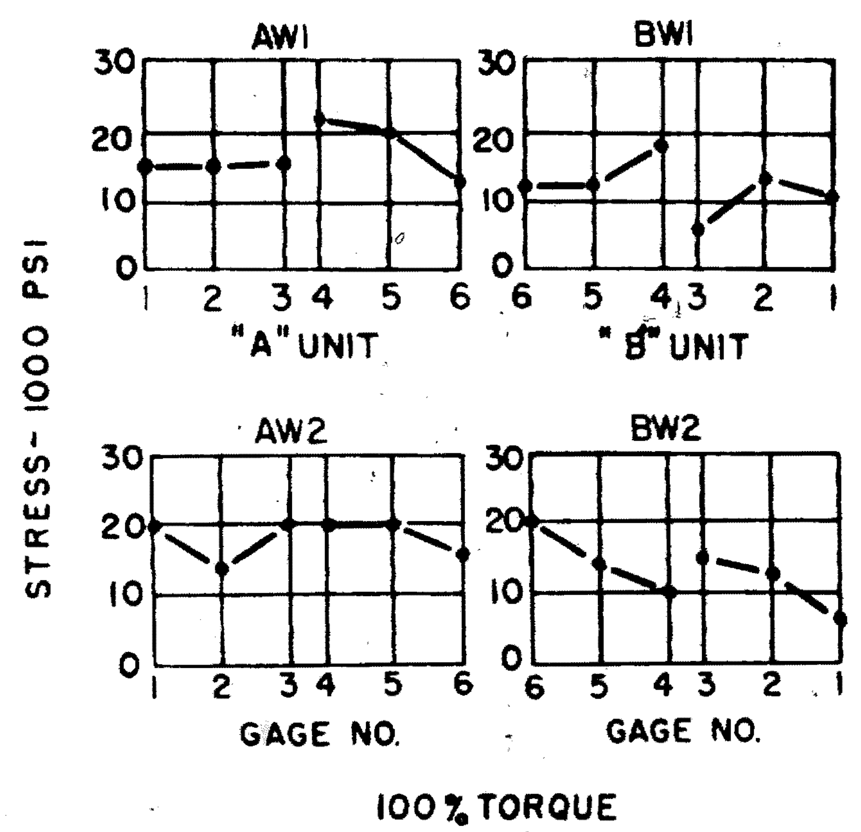


Fig. 5 Full load, full speed—strain-gage readings

Table 1 Gear data, full-power values

	Pinion		Gear
Pitch diameter, in.	12.009		42.261
Face width, each helix, in.		10.75	
Face width, total plus gap, in.		24.00	
Number of teeth	52		183
Diametral pitch		4.33015	
Normal diametral pitch		5	
Helix angle		30°	
Pressure angle		16° 37′ 22″	
Normal pressure angle		14° 30′	
Addendum, in.		0.200	
Dedendum, in.		0.2815	
Rpm		Over 4500	
Pitch-line velocity, fps		Over 250	
K-factor	174		
Tooth contact stress, psi	56,600		
Tooth bending stress,[a] psi	13,300		

[a] Calculated per Military Specification MIL-G-17859.

The excellent tooth geometry and alignment is proved by the set of strain-gage records taken at full load and full speed, Fig. 5. Note that there are data for both A and B units, and the instrumentation yields separate data for each mesh in each locked-train gear.

The variation in tooth load along the helix is not large, and the data also show that the load is nearly perfectly divided between helices and also between meshes, the latter indicating good "timing" of the coupled locked-train gears.

The expected good results were attained when the load was increased in 25 per cent steps to over 200 per cent torque. Fig. 6 gives the strain-gage readings at 206 per cent torque. A plot of the average of the six gage readings for many tests at various loads, Fig. 7, shows that the gage readings are proportional to torque, as would be expected with properly operating gearing.

Shown in Fig. 7 are curves labeled "maximum bending stress,"

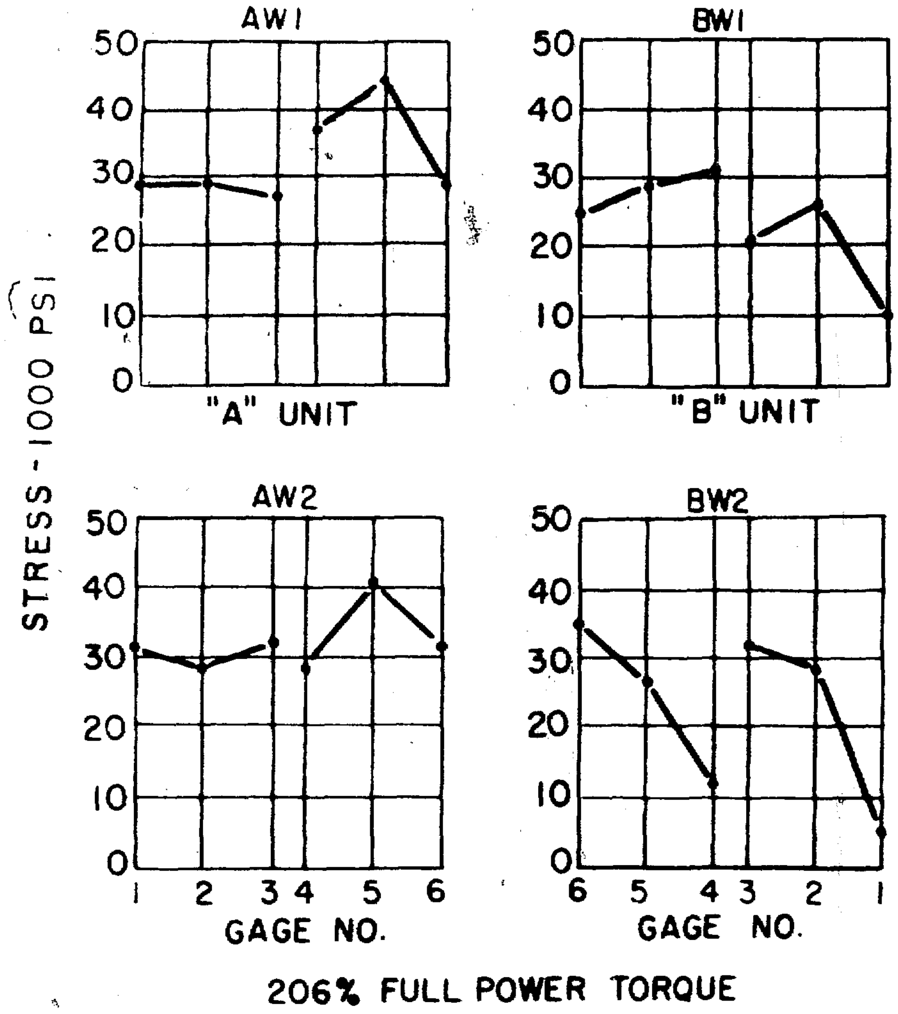


Fig. 6 206 per cent torque, full speed—strain-gage readings

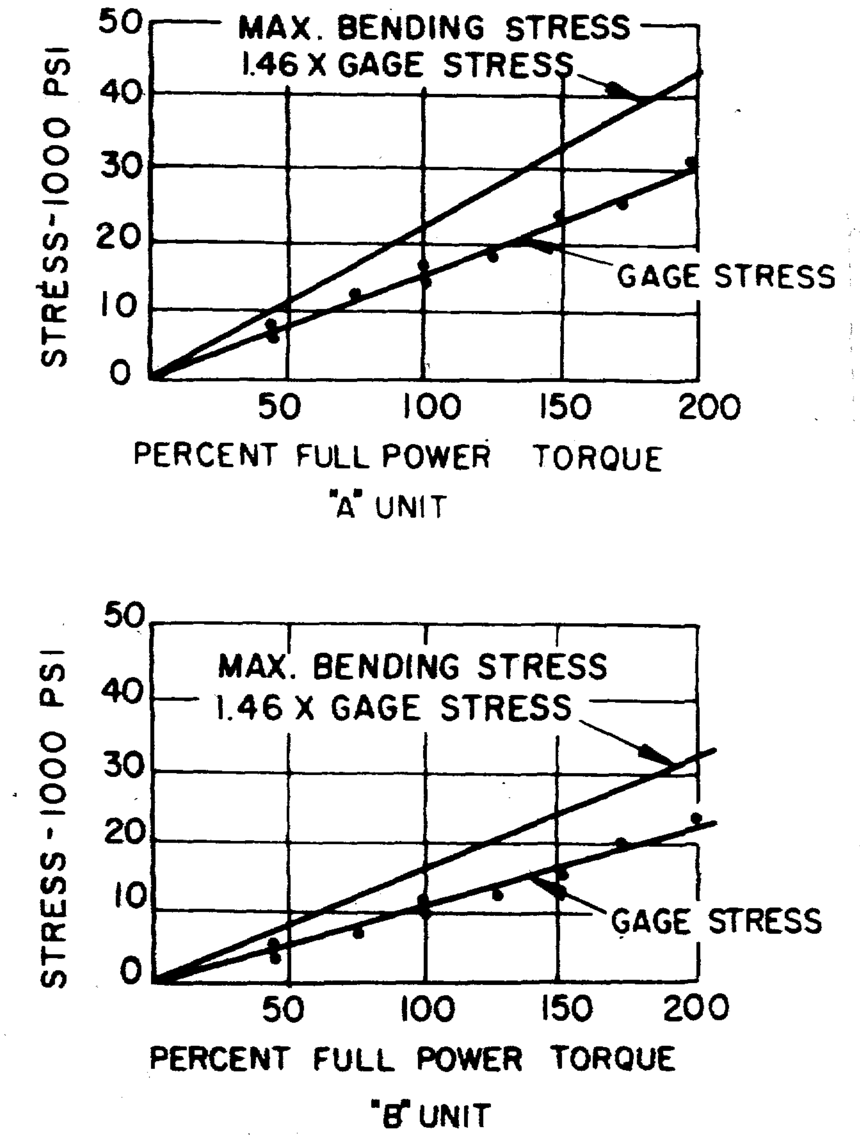


Fig. 7 Strain-gage stress versus torque or load

obtained by multiplying the gage reading by a factor (1.46) to account for the fact that the gage is not at the point of maximum stress and also is recessed from the surface.

Each of the load points shown in Fig. 7 was run continuously for 60 hr. This time imposes 5,000,000 cycles of stress on each gear tooth and 35,000,000 cycles of stress on each pinion tooth. This is sufficient to produce a fatigue failure if the bending stress exceeds the endurance limit.

After the conclusion of the extensive testing already described, comprising nearly 400 hr at loads up to 206 per cent rating, the pinion and gear teeth were in excellent condition, with no signs whatsoever of distress or wear, no pitting and no magnaflux indications of cracks.

Profile Effects. Although the pinions in the A and B units are alike in involute profile, the profiles of the gear teeth on the two units are different. The B-unit gears have nearly perfect involutes, with virtually no tip and root relief. The A-unit gear profiles have appreciable tip and root relief, the magnitude of which increases slowly from the pitch line to the tip or root. The pressure angles of the gear and pinion profiles are matched.

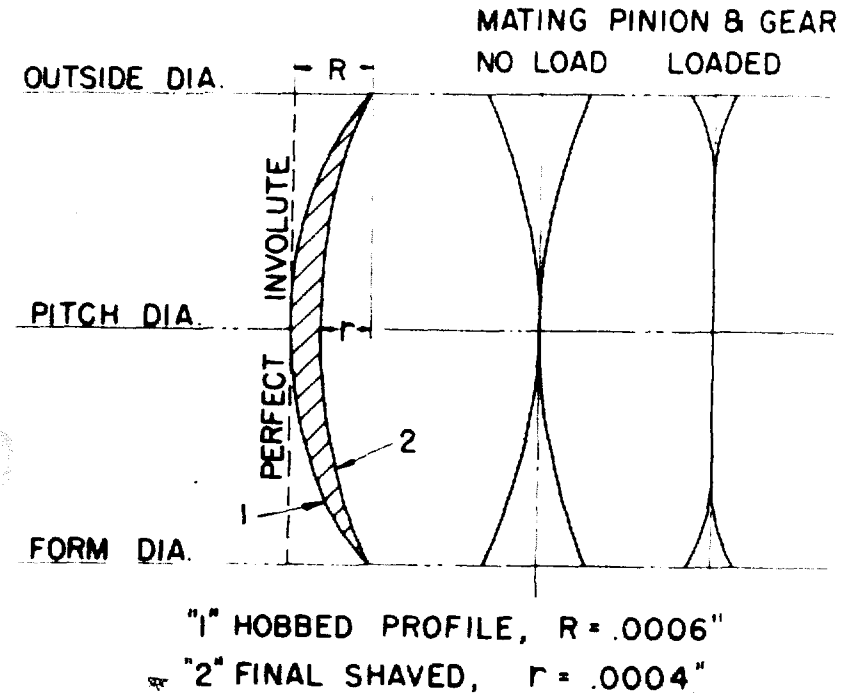


Fig. 8 Tip and root relief

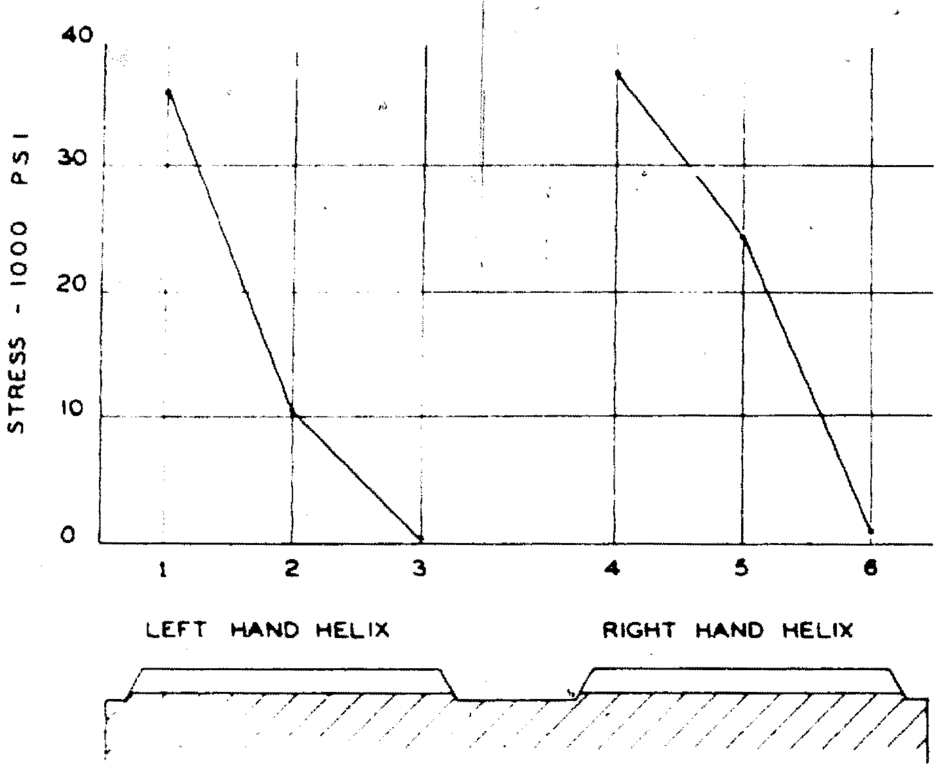
This type of tip and root relief is illustrated in Fig. 8, showing the deviation from a perfect involute for the pinion and gear and the resulting match of the two when underload. This type of relief contrasts sharply to the more sudden, abrupt, tip and root relief often employed by others.

It has long been believed that large amounts of tip and root relief are highly undesirable, the result being the equivalent of excessively stubbing the teeth and thus lowering the contact ratio and increasing both bending and contact stresses. The effect on bending stresses is shown by comparing the strain-gage readings for the two gear units in Fig. 5. The B unit, having little tip and root relief, shows an average gage reading of 12,400 psi. The A unit, with moderate tip and root relief, shows a much larger average gage reading, namely, 17,400 psi, which is 40 per cent greater than the B-unit readings. It should be emphasized that large tip and root relief values could easily double the tooth bending and compressive stresses.

Misalignment Effects. It has long been suspected that the over-all flexibility of gear teeth is of the order of twice that calculated from simplified methods of analysis. The results of many tests conducted on the laboratory gears support this conviction and demonstrate that gearing of this type can accept moderately large misalignments without danger.

Illustrative of these data is Fig. 9, showing a full-power run with one gear bearing intentionally raised 0.010 in. Since the properly timed locked-train gear distributes misalignment equally between the two meshes, this corresponds to 0.005-in. misalignment across one mesh, or 0.002 in. across a single helix (which is 11 in. long).

Fig. 9 shows that this 0.002-in. misalignment drove the level tooth loading into a triangular shape across the face width. This effectively doubles the bending and contact stresses at one end of



MISALIGNMENT EFFECT ON STRESS DISTRIBUTION

Fig. 9 Intentional misalignment of 0.010 in.

each helix and lowers these to zero at the other end. Although this is an undesirable operating condition, the gearing material has adequate reserve to withstand indefinite operation under triangular loading distribution across the complete face width. Thus a 0.002-in. misalignment across a single helix is not a totally unacceptable operating condition, for gearing of the type considered and comparable loadings.

These data further permit the quantitative conclusion that corrections for pinion deflection due to bending and twist are entirely unnecessary when the mismatch due to these deflections is 0.001 in. or less. With more highly loaded gearing, corrections for these deflections become even less important, provided equal nominal factors of safety are maintained by the use of higher strength materials.

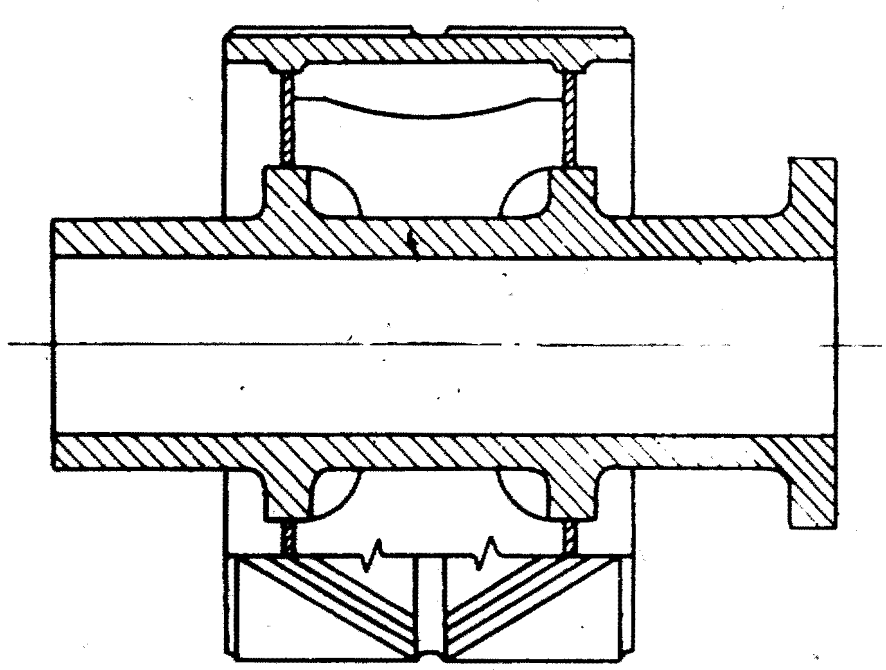


Fig. 10 High-speed gear wheel

Speed Effects. The construction of the high-speed wheels, Fig. 10, permits unequal expansion of the gear rim due to centrifugal forces as caused by speed. For the wheels under test, the differential expansion from high to low point along the length of the tooth is calculated as 0.0022 in., at rated full speed.

The result of this differential expansion is seen in Fig. 11, showing the loading diagrams for $^1/_4$, $^1/_2$, $^3/_4$, and full speed. The effect is seen to be distinct but not large.

Center-Gap Cooling. Without special center oil sprays the centers

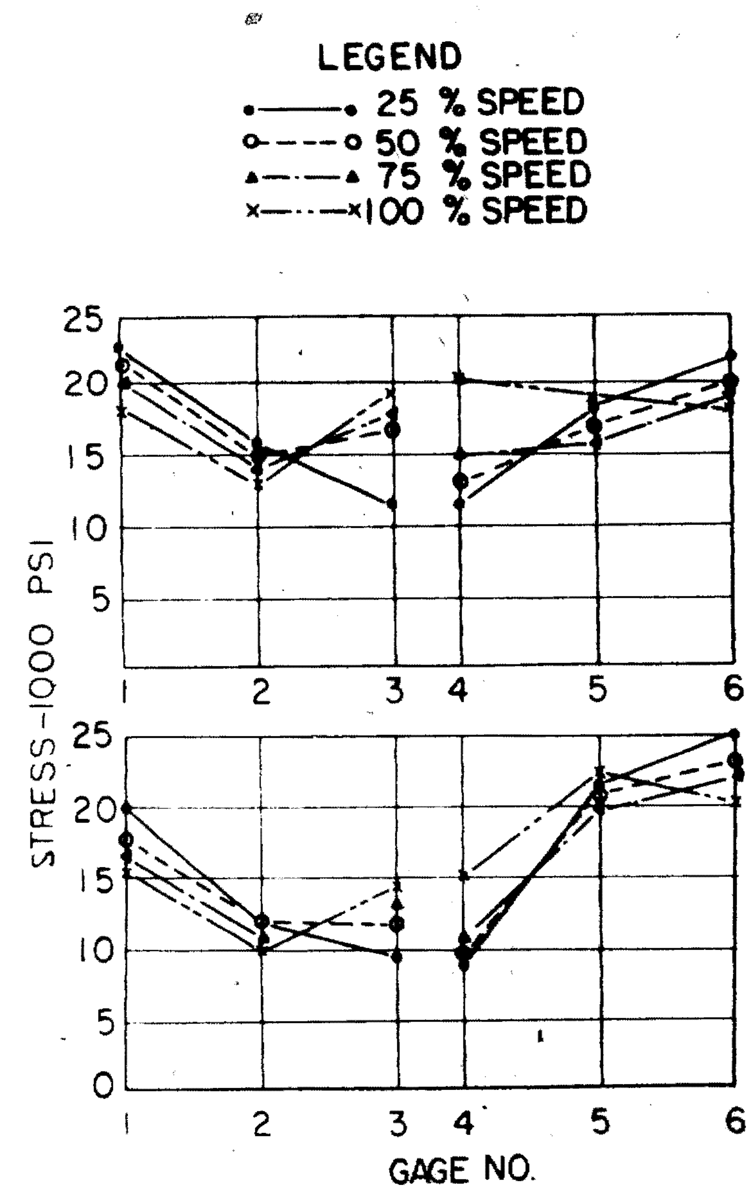


Fig. 11 Effect of speed on stress

of the pinion and gears run at higher temperatures than the ends or edges, the resulting differential thermal expansion producing a "barrelling" of both pinion and gear and consequent higher tooth loads near the gap. By valving the center sprays on the A unit, the beneficial effect of center-gap cooling is demonstrated while the gears are operating without any interruption or shutdown. The results for 100 per cent load, shown in Fig. 12, are again significant but not large. Shutting off the center sprays causes the stress at the gap to increase approximately 30 per cent.

Oil-Spray Location. The basic arrangement of sprays for the A unit consists of four mesh sprays and one center spray for each mesh, spraying the ingoing mesh. Many variants of sprays have been tried, including: (*a*) Sprays on the ingoing side for one mesh and sprays on the outgoing side of the mesh for the other mesh, and (*b*) only one bank of sprays, but including a center spray. Despite the many variants tested, none produced a significant change in load or stress distribution, except when the center sprays were entirely cut off, as previously described.

Summary. Although important quantitative results have been obtained with these tests, the gearing as designed and manufactured displays excellent load-carrying abilities and could reasonably be expected to run without failure or distress at considerable overloads and undermarked distortions due to uncontrollable deflections and misalignments.

Shop Test Results

The modern trend for highly loaded naval propulsion gears is to full-power test each production gear for periods ranging up to 24 hr. Our latest test-floor arrangement for conducting these endurance tests is shown in Fig. 13. A special test gear having an integral loading device is arranged for convenient connection to the production gear, in a back-to-back arrangement. The small drive turbine operates the test at rated speed and also is used to conduct the overspeed test.

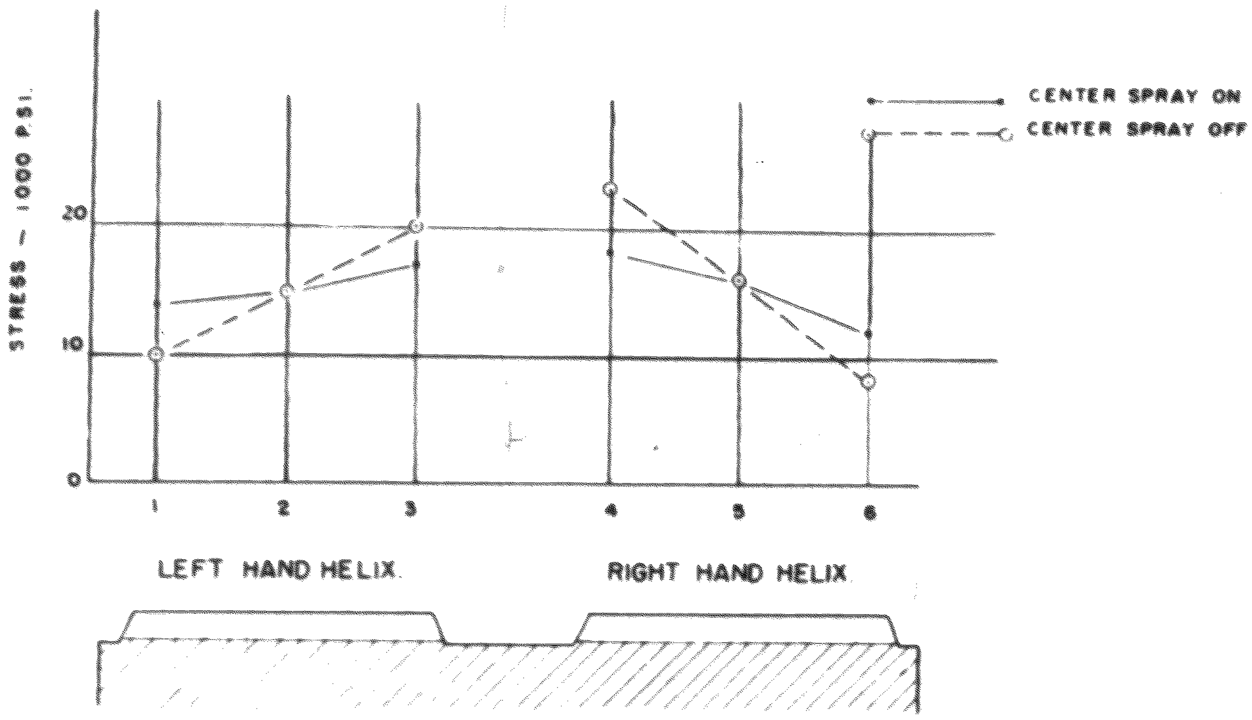


Fig. 12 Effect of center spray on stress distribution

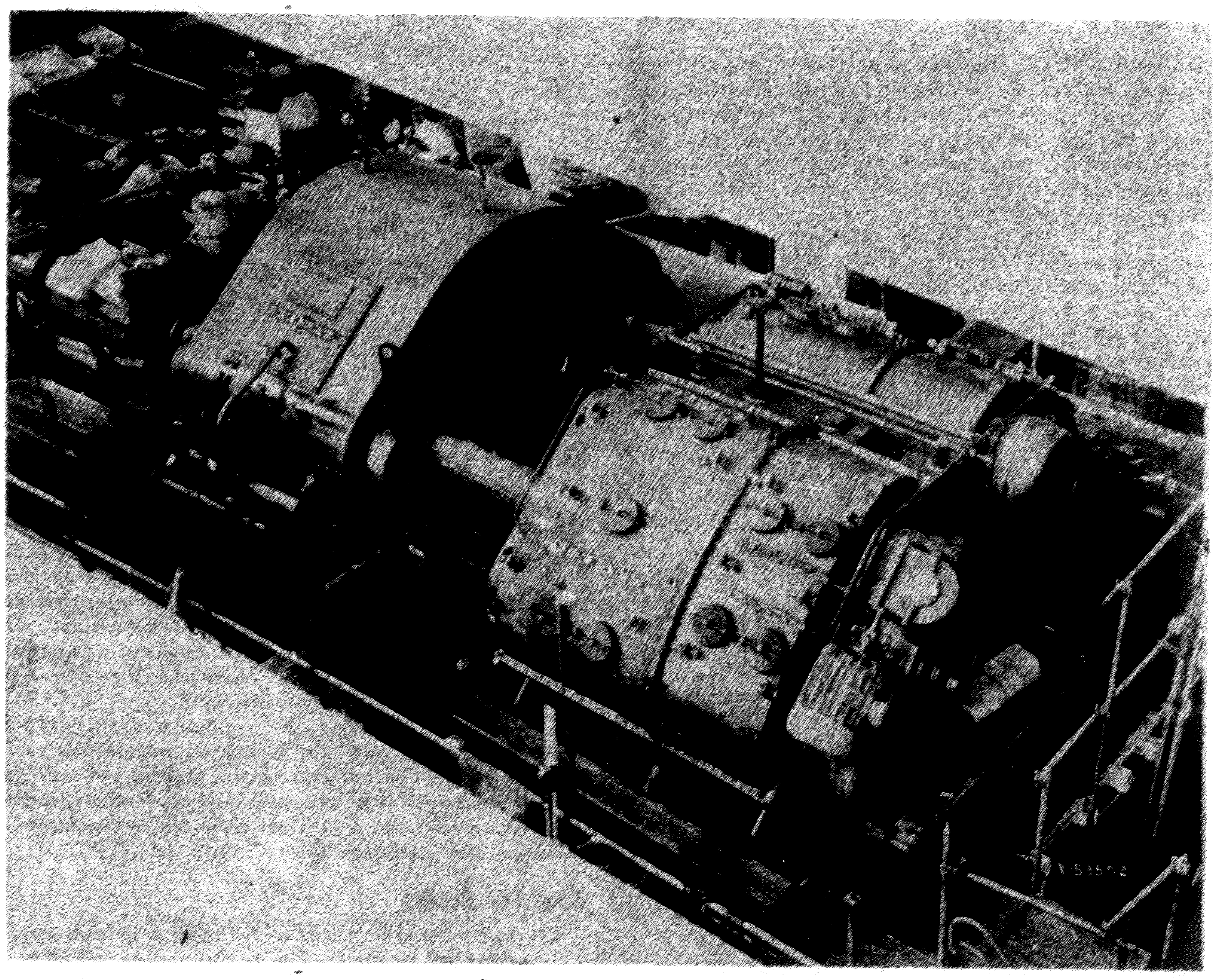


Fig. 13 Test floor arrangement

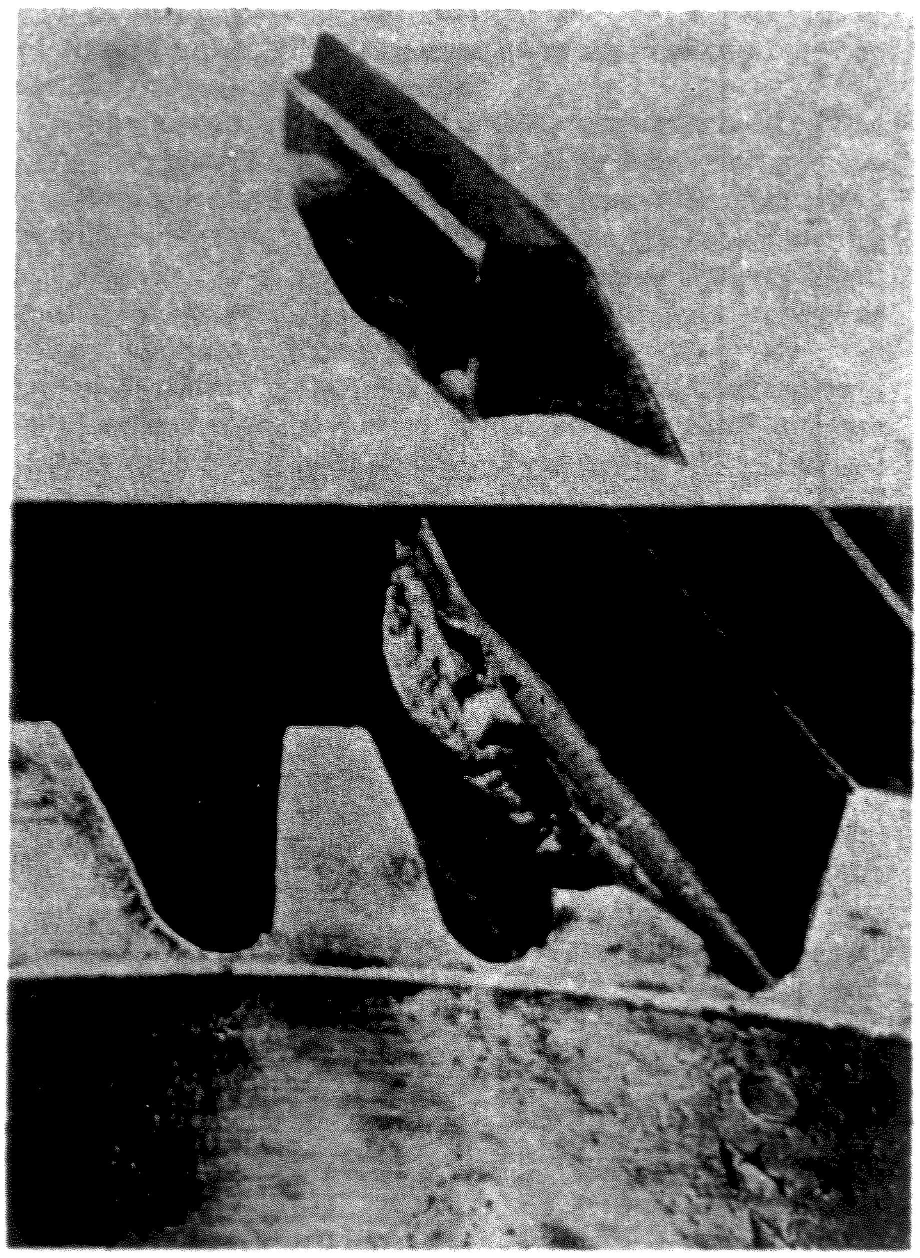


Fig. 14 Tooth failure (located at gap end of aft helix)

A recent order for large propulsion gears had tooth geometry and loading nearly identical to that used in the laboratory test and described in Table 1. The first two gears successfully underwent a 24-hr full-power shop test, with no difficulty or distress. The third gear experienced a tooth failure on the high-pressure high-speed pinion, after 12 hr of full-power operation.

This failure consisted of a fatigue fracture at the gap end of the aft helix of the pinion, a length of tooth approximately 1 in. long having broken out, as shown in Fig. 14. The fracture started from the face on the inside of the V or apex formed by the right and left-hand helices, as shown at x in Fig. 15. An unusual aspect of the failure is that the crack started on the active profile slightly above the first point of contact in the dedendum, rather than in the root at the point of maximum bending stress.

Close examination of all teeth during subsequent marking runs revealed several significant facts:

1 The helical angle match between pinion and gears and the alignment were excellent as measured by slow roll marking at two thirds of full-load torque.

2 The full-speed, high-torque marking was very good when the teeth were loaded with the load on the outside of the V (or apex) on the pinion, Fig. 15(*a*).

3 The full-speed, high-torque marking was very poor when the teeth were loaded with the load on the inside of the V (or apex) on the pinion, Fig. 15(*b*). The marking was distinctly heavy near the gap ends of the teeth, and distinctly light near the flank ends of the teeth, Fig. 15(*c*).

4 Accompanying this uneven, gap-heavy loading, the gear teeth showed excessively heavy marking in the dedendum near the last line of contact with the pinion.

Since the gear under test was a two-turbine-input locked-train gear, there were a total of four high-speed wheels on which the 2nd, 3rd, and 4th observations could be made. The consistency of these observations was remarkable, pointing strongly to a phenomenon not yet explained.

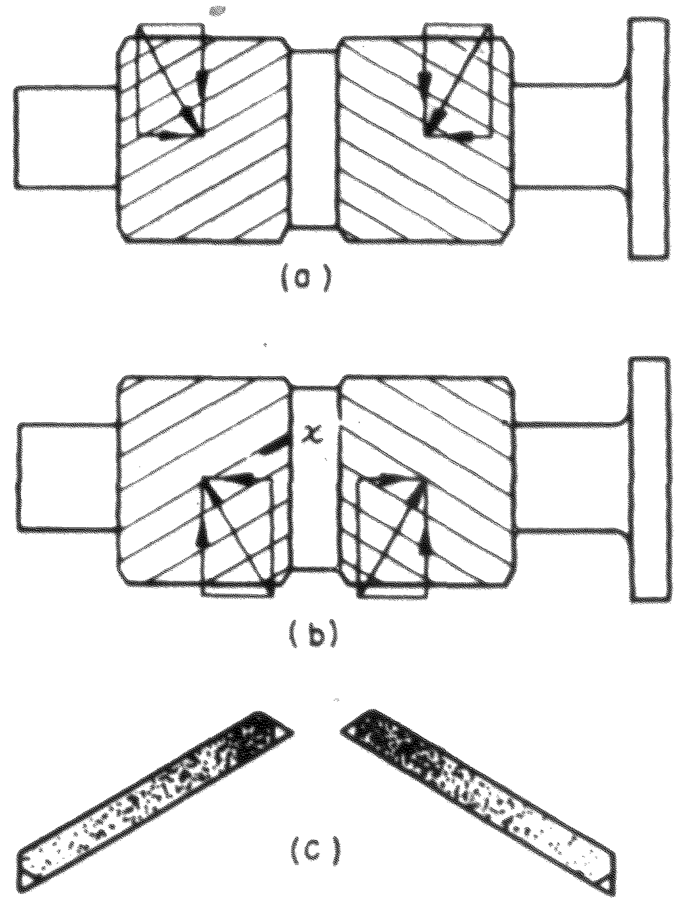


Fig. 15 Direction of loading and gap-heavy marking

It soon became evident that the problem was a thermal one, since it was discovered that immediately after a run with loading in the adverse direction, the center of the pinion was much hotter to the touch than the ends. Improved methods of cooling were introduced, and a routine was set up to obtain contact thermometer readings on the centers and ends of the gears and pinions immediately after a fast controlled shutdown, so as to evaluate the effectiveness of the improved cooling.

These data are presented in the curves of Fig. 16. When extrapolated backward to zero time, these curves show positively that only when the loading is on the outside of the V of the helices does the barrelling thermal effect show up strongly on the gears. The effect is always present to a noticeable degree on the pinions; however the same temperature differential on the wheels is many times more serious since the diameter of the gears is three times that of the pinion.

Explaining Fig. 16 in more detail, the back-to-back test in Fig. 13 loads one side of the ship gear ahead and the other side astern. Figs. 16(*a*) and (*b*) correspond to this. Then the load is reversed in direction, reversing the torque on the two sides, for which Figs. 16(*c*) and (*d*) apply. In each case, only one side shows adverse thermal effects on the gears, the side being different in the two cases, Figs. 16(*a*) and (*d*), and both sides showing adverse loading had been torqued so that the load was on the *inside of the V of the pinion*.

As a result of these observations, the thermal effects were examined more closely and a theory developed that will be described.

Thermal Instability of Gearing

The heat generated at the mesh, although a small part of the power transmitted, must be conducted away and radiated from the area of generation at a sufficiently high rate to prevent thermal instability. This is a problem that becomes critical only when the power flux through the gear is high. Thus the problem is limited, for the most part, to high-horsepower-per-mesh gears, and these perforce are gears with high or moderately high pitch-line speeds.

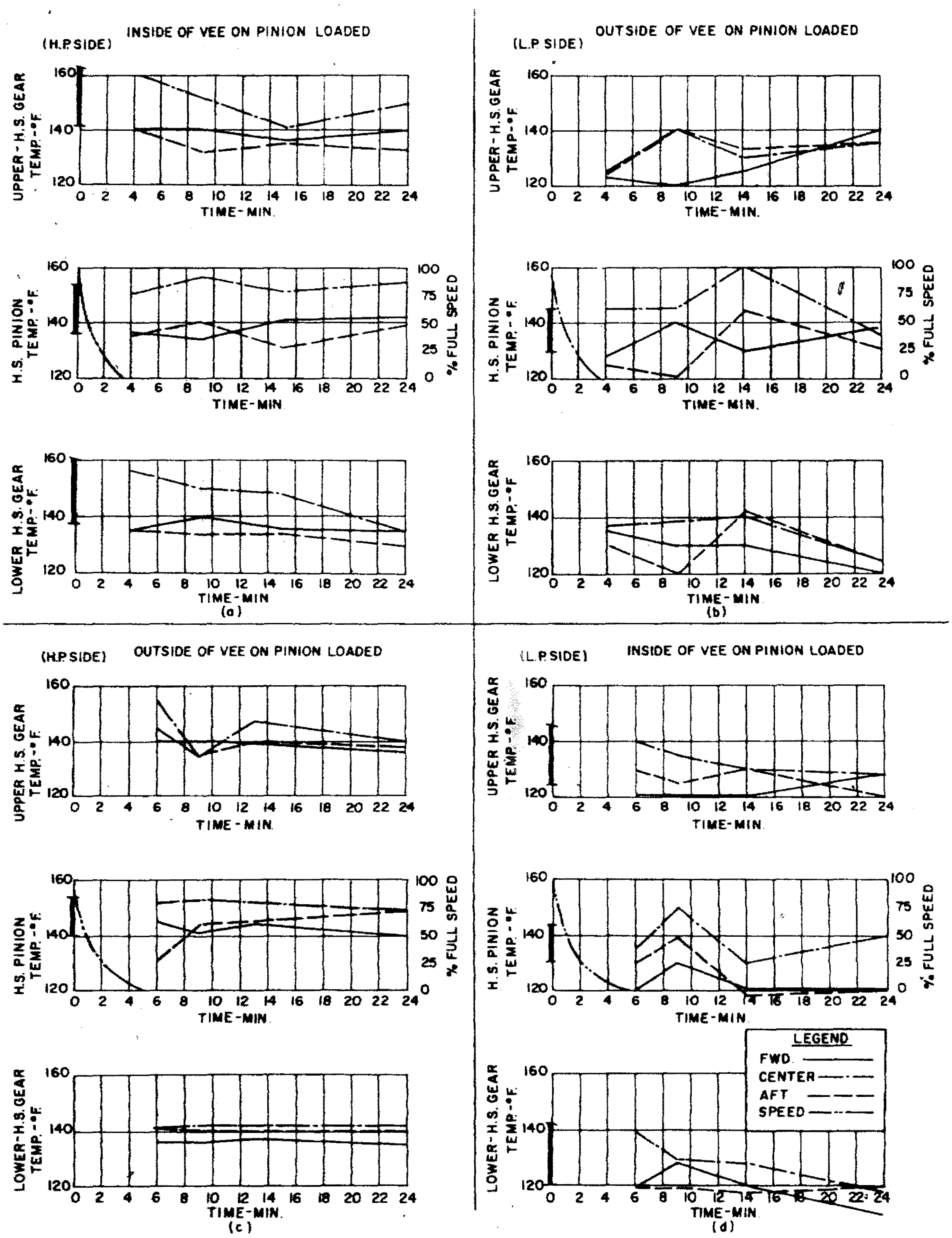


Fig. 16 Temperature-time curves

The heat loss at the mesh has never been accurately determined for large helical gears, but by deducting the known bearing losses from the over-all losses of a reduction gear under full-power test, the mesh losses are found to be of the order of 0.2 per cent of the power transmitted. The distribution of the generation of heat during the action of the gear and pinion teeth from the first point of contact to the pitch point and thence to the last point of contact is illustrated in Fig. 17, from a recent paper by Shipley [5], who experimentally determined the average coefficient of friction to be approximately 0.04 for spur gears operating at pitch-line speeds up to 100 fps.

A crude first approximation is that the average coefficient of friction does not change with pitch-line speed or load. The power loss is then proportional to: (*a*) The number of engagements per

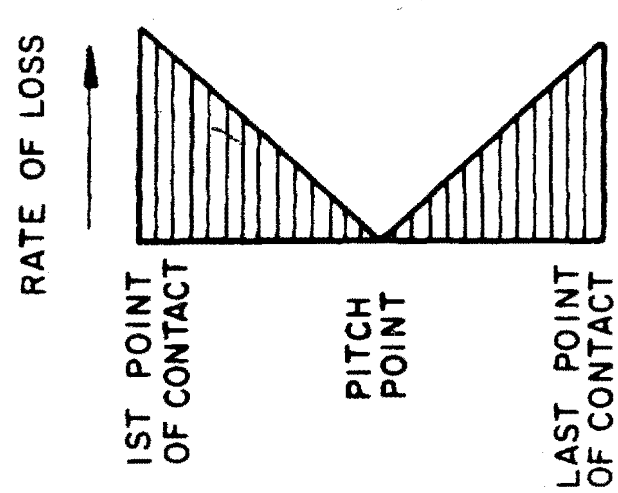


Fig. 17 Heat loss in gearing

second, or pitch line velocity; and (b) the total tooth load, or transmitted torque. Thus the *power loss* is proportional to the power flux or *transmitted horsepower*.

The heat generated is conducted from the mesh area by the body of the pinion to the bearing journals, and by the rim of the gear wheel to the side plates, center plates, and hub. Heat is removed by convection via the cooling oil from the sprays and by the turbulent air in the gear case. The radiation effect is a small percentage of the whole due to the relatively low temperatures of the pinion and gear surfaces.

Any temperature differences upset the accurate geometry of the gearing and cause abnormal effects, some of which increase the rate of heat loss at local sections of the mesh.

Barreling of Pinion or Gear. The center of a pinion or gear will almost inevitably be hotter than the ends, since the center plane is a plane of symmetry and the heat flow will be away from this plane, on both sides, unless unusual methods of cooling the center are employed. The resulting differential thermal expansion of the center of the pinion or gear produces a "barreled" shaped pitch cylinder, Fig. 18, which leads to greater load per inch of face width near the gap and a lesser unit load near the ends.

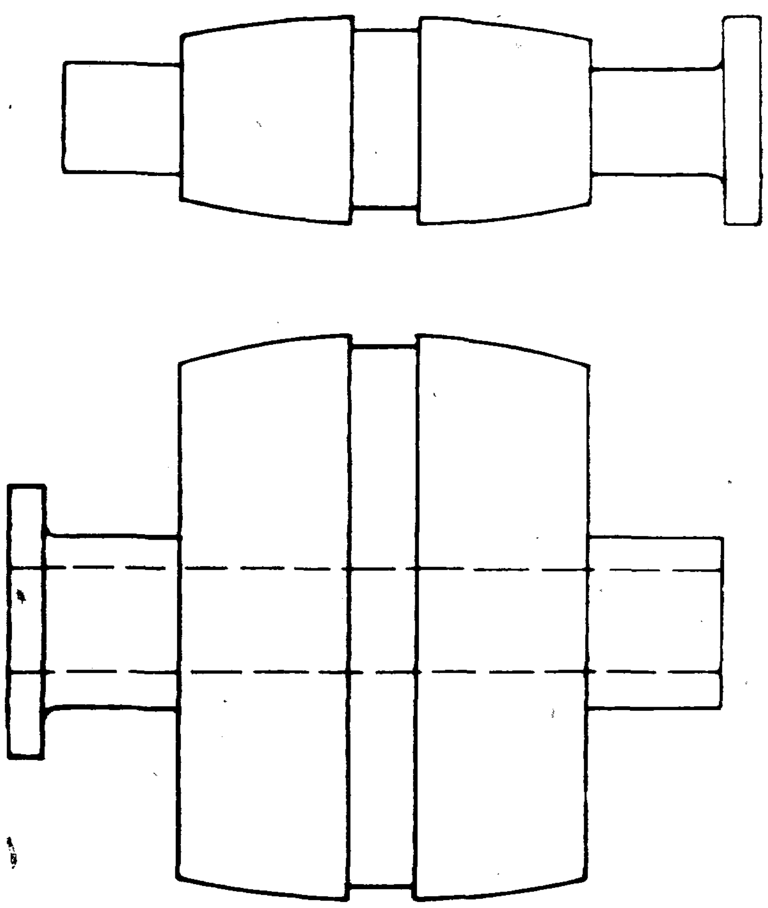


Fig. 18 Barreling of pinion and gear

The local effect on tooth load is proportional to the relative thermal expansion or barreling, the pressure angle in the plane of rotation ϕ, and the tooth flexibility. The effect is independent of helical angle. For evaluation the pinion in Fig. 19 will be considered as having spur teeth and it will be assumed that a 1-in. length near the gap is expanded in a stepwise manner, 0.0002 in/in. which corresponds to a temperature differential of 30 F in steel. Fig. 20 shows the effect of this expansion on the tooth position in the local section. The profile is made "proud" at each point by

$$r\epsilon \sin \phi$$

where r is the radius to the point on the profile, ϵ the unit expansion (e.g., 0.0002 in/in.), and ϕ the pressure angle at the point in question. At the pitch point on the pinion whose dimensions are shown in Table 1

$$r\epsilon \sin \phi = 6 \times 0.0002 \times 0.286$$
$$= 0.00034 \text{ in.}$$

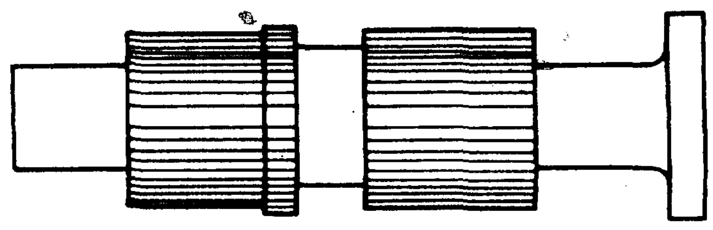


Fig. 19 Expansion of 1-in length of pinion

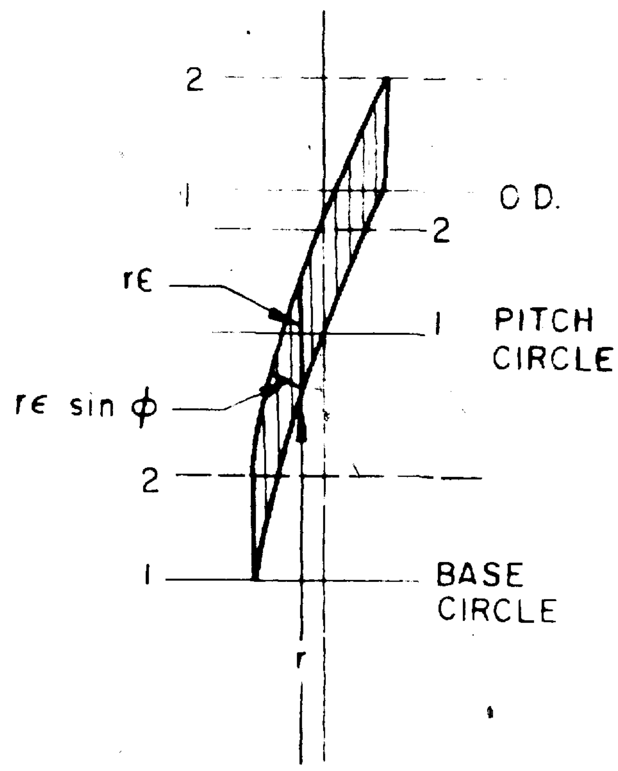


Fig. 20 Effect of expansion on tooth profile

and the gear

$$r\epsilon \sin \phi = 21.13 \times 0.0002 \times 0.286$$
$$= 0.00121 \text{ in.}$$

both values being for a 0.0002 in/in. expansion (ΔF = 30 deg). The combined effect would be

$$0.00034 + 0.00121 = 0.00155 \text{ in.}$$

which is very significantly large.

A more subtle effect is the influence of pinion expansion on contact up and down the profile. As may be seen in Fig. 20, the tip of the pinion will distort more than the root. This is due to the greater radius at the tip and also the fact that the pressure angle is greater at the tip, the amount of heavy contact being given by the equation:

$$r\epsilon \sin \phi = r\epsilon \sin \cos^{-1} \frac{r_b}{r}$$

where r = radius to any point on the profile and
r_b = base circle radius

If the pinion only is expanded locally, Fig. 19, the profile is "proud" by the following amounts (for the pinion of Table 1):

First point of contact: $r\epsilon \sin \phi$ = 0.00022 (root of pinion)
Pitch line: $r\epsilon \sin \phi$ = 0.00034 (pitch point)
Last point of contact: $r\epsilon \sin \phi$ = 0.00045 (tip of pinion) again for ϵ = 0.0002 in/in.

Observations of tooth contact after full-power operation, on gears known to have excessively warm pinions, clearly show the strong root-heavy marking on the teeth of the mating gear wheel, as predicted by the theory in the previous paragraph.

Effect of Temperature Difference Between Pinion and Gear. In most high-speed gearing, the pinion will have a higher average temperature than the mating gear. The reasons are twofold: (a) The number of engagements per second per tooth is higher, so

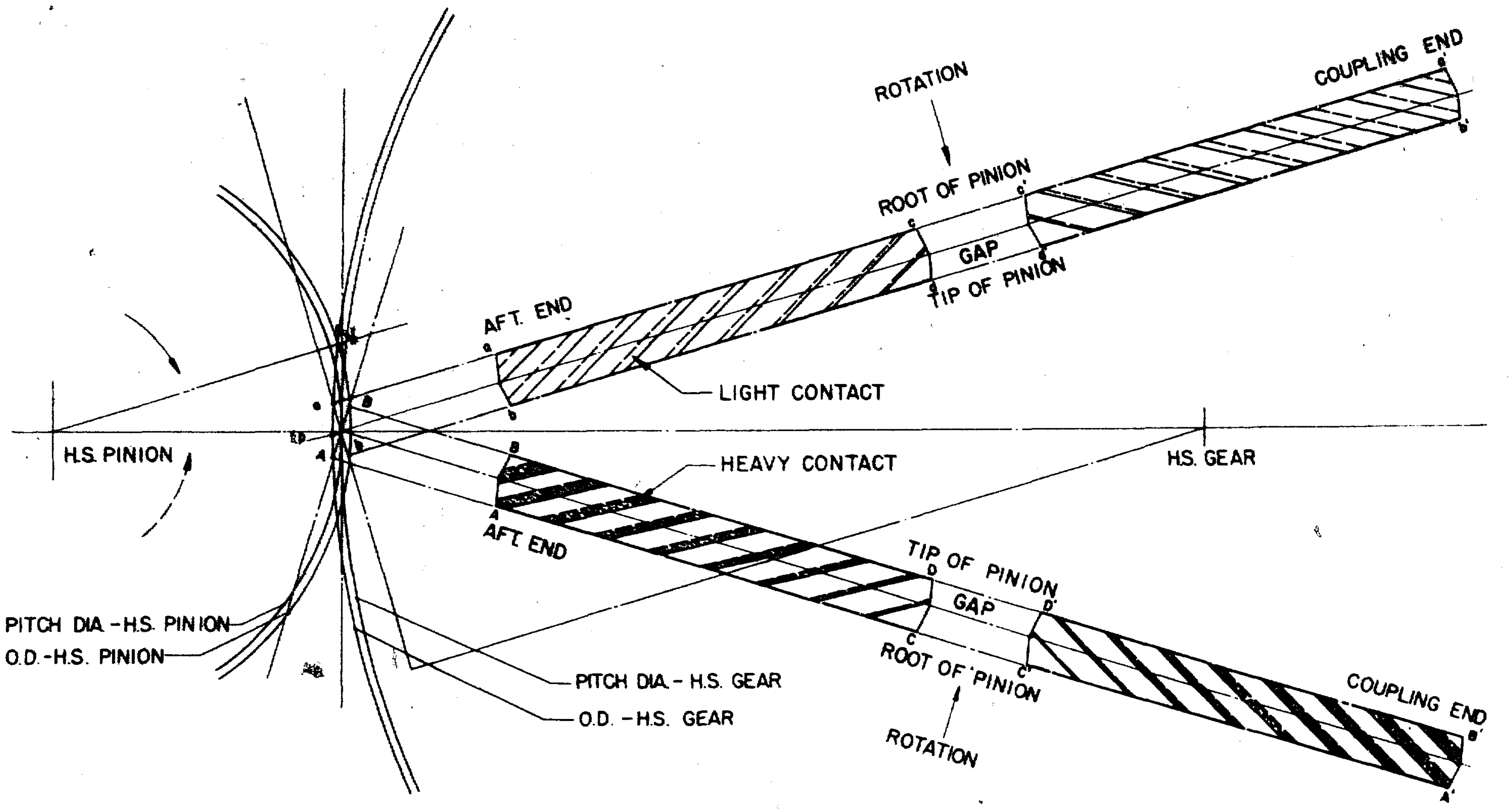


Fig. 21 Effect of temperature differential between pinion and gear

that the heat input per unit surface area is higher on the pinion by a factor equal to the gear ratio; and (b) the cooling of the wheel is superior to that of the pinion. On a locked-train gear, where one pinion engages two gears, the first reason is doubly important.

A uniform temperature difference of, say, 30 F between pinion and gear would not produce significant effects on a spur gear, the sole effect being to create slight tip-heavy loading on the pinion. On a double-helical gear, however, the effect would be most significant.

Fig. 21 is a pitch-circle layout of the pinion and gear in Table 1, showing the base circles, the outside diameters, and the line of action apb. The plane of action, determined by apb, is projected into the plane of the paper to form the rectangle $abcd$ for the aft helix and $a'b'c'd'$ for the forward helix. The contact lines between pinion and gear are shown in this plane, these lines being straight lines inclined to the axis at the base-cylinder helical angle. It should be noted that the forward helix (on the pinion) is a right-hand helix, whereas the aft helix is a left-hand helix, this being established conventional practice in some activities.

The line of action shown is for the pinion driving in the direction of the solid arrow, which is the standard direction for a single-screw ship, or the starboard propulsion plants of a multi-screw ship.

Now consider the effect of a uniform temperature differential between pinion and gear, the pinion being the hotter. If both were made of completely rigid material, so that only thermal expansion and no elastic deflection occurred, the following would result. The pinion would grow uniformly in every direction, and, if one point (say, d) is taken as a reference point, the thermal expansion would be proportional to the distance away from the reference point. Points d and d', on the last teeth in contact, would remain in contact. The other pinion teeth would progressively back off from the mating gear teeth by the distance between the dotted lines and the original contact lines, this being the amount of the thermal expansion. Thus, axially from d to b, the loss of contact increases to a large amount at b, although along the line of action d to c, the separation is relatively small. A duplicate effect occurs on the forward helix, where point d' stays in contact and the other pinion teeth that were previously in contact back away from contact by an amount proportional to the distance away from point d', measured in the plane of action.

The pinion and gear, being made of elastic rather than rigid materials, do not lose contact as shown. Instead, the teeth tend to *hog the load strongly adjacent to the gap, and shirk the load strongly near the outside ends.* For future reference we shall call this situation Case I. A more general viewpoint of this effect can be obtained if one considers that the pinion, being hotter than the gear, expands uniformly in all directions. The imaginary but important base cylinder expands uniformly, altering both the base pitch and the axial pitch. Altering the base pitch causes the slightly heavy tip loading, and altering the axial pitch causes the much more important effect of creating gap-heavy, end-light, loading.

Now examine this effect when the pinion drives in the opposite direction, as shown by the broken arrow. This is the direction of rotation for port units on multiscrew ships. The line of action is now ApB and the plane of action the rectangle $ABCD$. The hand of the helices is the same as before, since the gearing geometry has not been changed.

Now when the pinion is uniformly hotter than the gear, and points C and C' are taken as the reference points for the two helices, a converse effect occurs. Progressing away from the point C, the pinion teeth expand *into* the gear teeth by a progressively larger amount. Thus the gap ends of the teeth tend to *shirk* the load and the *outside ends tend to hog* the load. This will be called Case II.

It should be carefully noted that the difference between the Case I effects and the Case II effects is determined by the direction of loading and not by the direction of rotation. Case I occurs when the pinion teeth are loaded on the inside of the apex, Fig. 15(b). Case II occurs when the tooth loading on the pinion is on the outside of the apex, Fig. 15(a). On the gear wheels, of course, the inverse loading occurs.

Conditions Leading to Thermal Instability or Regeneration. Consider the local effects of a thermal expansion as shown in Fig. 19. The tooth load in this local section increases markedly. This increases the heat generated in the mesh in this local section, thus raising the temperature of the local section, increasing the expansion and hence the load and the heat generated. Meanwhile the unit loads in other areas are being slightly decreased, and thus these other areas are slightly contracting. It is not difficult to conceive that this process can be thermally unstable or regenerative, to the extent that the process will continue until a tooth failure occurs at the local section. What then are the conditions that may lead to a regenerative type of failure?

Case I as described is the condition when the loading is on the inside of the apex of the pinion. It was shown in Fig. 21 that this leads to gap-heavy tooth loading. Heat flows with great difficulty from the center of the pinion and gear. This case then has the basic elements of a regenerative or unstable system.

Consider Case II loading, with the load on the outside of the apex of the pinion. This leads to an end-heavy, gap-light tendency. The cooling conditions on the ends of the pinion and gear are good, both by conduction and by convection. This case then has the basic elements of a degenerative or stable system.

Now add the barreling effect, as caused by a temperature difference along the length of the pinion, which not only normally occurs to a limited extent, but which will be amplified by the warm-pinion effect. This barreling effect is *directly additive* to the Case I effect, adding load at the gap and subtracting load at the ends, Fig. 22. Thus the barreling effect increases the unstable tendency of the system for Case I.

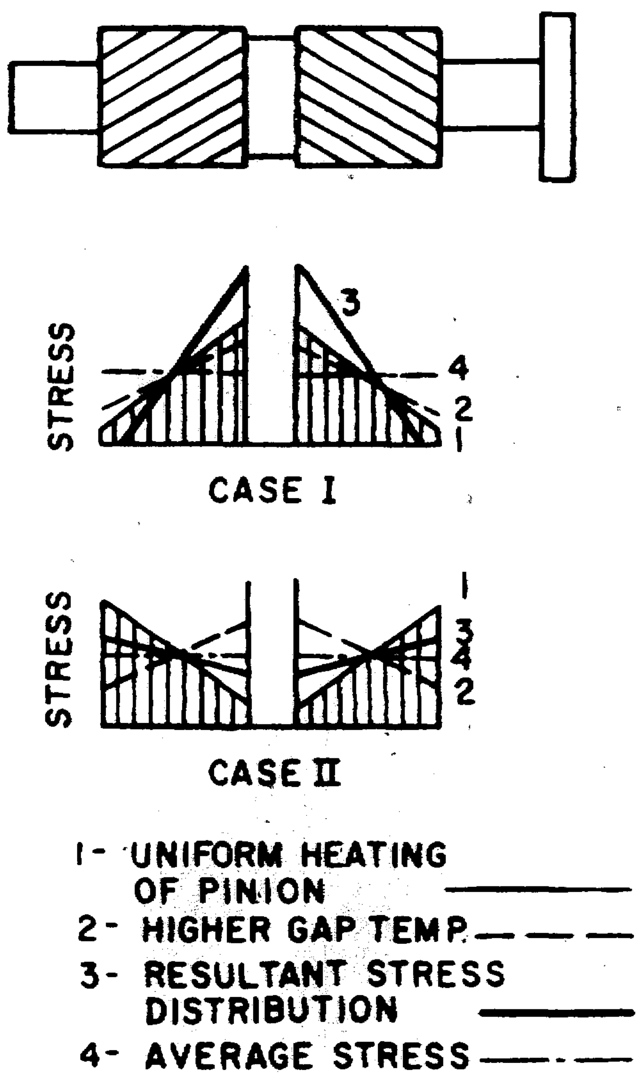


Fig. 22 Combined effects of temperature differentials

The influence on Case II of the barreling effect is subtractive, decreasing the concentration of loading at the ends and supplying additional loading to the gap-light sections of teeth. Thus a system that tends basically to be stable or degenerative is further aided by the barreling effect.

The composite effects are shown graphically in Fig. 22, and to summarize the foregoing conclusions:

On a double-helical pinion and gear, thermal instability leading to tooth failure or profile distress can exist if the pinion is loaded with the load on the inside of the apex of the teeth. When the loading is on the opposite side of the teeth (i.e., *on the outside of the apex*), *thermal instability is most unlikely to occur, the system being basically stable.*

Unfavorable Laboratory Results

After the theory in the preceding section had been developed, it became self-evident that the laboratory test up to then had been operated in the *favorable* direction of loading. We immediately set about to repeat the experiments with the direction of loading reversed. For this loading and the same direction of rotation as before, the A unit in Fig. 4 is acting as a speed increaser, and the B unit as a reduction gear of identical rotation to that on a ship's starboard shaft. The first tests were run at 100 per cent load.

The stress distribution curves in Fig. 23 show at once that the loading is gap-heavy. On the A unit, although the coupling end helix (gages 4, 5, and 6) has essentially level loading, the opposite-end helix (gages 1, 2, and 3) show severe nonuniform loading, which is almost triangular in shape, nearly doubling the gap end stress. And this has occurred on a set of gears whose static alignment has been proved to be excellent!

If the upset of level loading on the A unit is classified as severe, then the result on the B unit should be called violent. Two out of four meshings on the two helices show violently gap-heavy loading, the remaining two showing a strong gap-heavy tendency. In two instances the stress at the gap has increased to 250 per cent of average value.

It should here be recalled that the B unit has little tip and root relief on the profiles, while the profile modification on the A unit is moderate in magnitude.

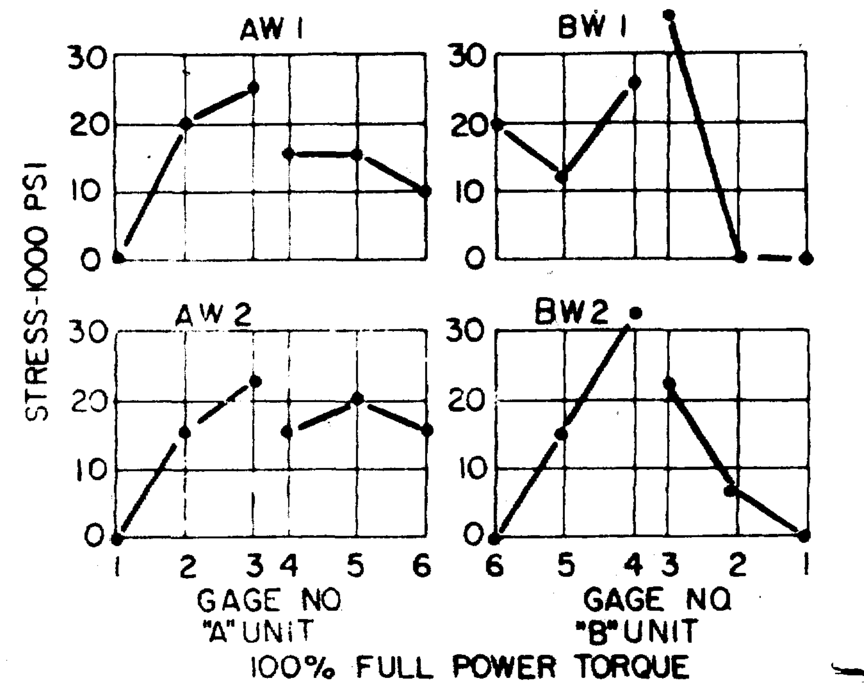


Fig. 23 Tooth stress at 100 per cent torque. Reversed loading.

The ordinates of the curves in Fig. 23 and the subsequent figure are not actual gage stress values but are these values multiplied by a factor to make the average values comparable in magnitude to the average values of the preceding curves. This procedure is required since the strain gages are on the loaded side of the teeth for the tests described in this section, and on the nonloaded side of the teeth for the favorable laboratory tests.

Further experimentation at 100 per cent load revealed the important information that the gap-heavy condition was accentuated by deviations in the oil-spray patterns, in contrast to the negative conclusions for the opposite direction of loading. Also when the center sprays were entirely cut off, the gap stress rose 70 per cent, whereas this rise was only 30 to 40 per cent for the opposite direction.

To corroborate these observed unstable effects, the torque was increased to 140 per cent full-load torque and ran for 6 hr at full speed. The curves in Fig. 24 show that now all meshes of both units have become gap-heavy. On the A unit, the worst case shows a triangular loading across the full face width, leading to a doubling of the stress at the gap.

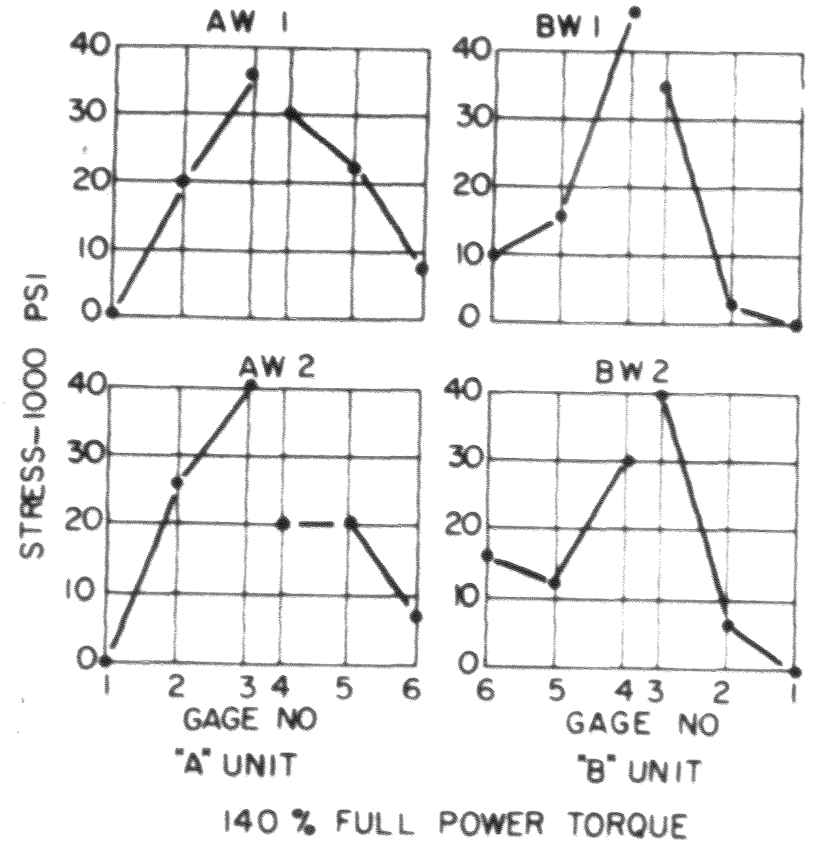


Fig. 24 Tooth stress at 140 per cent torque. Reversed loading.

On the B unit, however, the stresses at the gap have increased more violently, and it is evident that the noncoupling end helix is not loaded over more than 70 per cent of its face width. The peak stress reading is over 300 per cent of full-load average stress. The surface temperatures of the pinion were higher than usual, 182 F at the gap and 163 and 167 F on the ends. At 100 per cent load the corresponding temperatures were 173 F at the gap and 151 F at the ends.

Upon shutdown and inspection, a broken tooth was found on the pinion of the B unit, at the forward end of the aft helix. This fracture, shown in Fig. 25, is almost a perfect duplicate of the failure previously obtained during the shop test already described for a similar gear. The fracture occurred on a tooth not carrying a strain gage, so that the stress must have been slightly higher than the values given on the curves in Fig. 24.

In addition to the actual tooth failure and the strain-gage readings, heavy gap loading was shown on the copper-sulfate marking patterns on the gear teeth. Furthermore, the aft helix of gear BW 2 (see Fig. 4) showed a line of pitting in the lower dedendum adjacent to the gap, again this being the type of profile distress noted on the shop test.

In summary, the laboratory test gave most unfavorable results when the loading was reversed to duplicate the unstable condition predicted by the theory. At 140 per cent load a tooth failure was produced which was identical in every respect to a recent prior failure on a shop test of a large ship gear.

It should here be noted that all of the tooth-failure difficulties that we have experienced on highly loaded gears have occurred

Fig. 25 Pinion tooth fracture—"B" unit

in the shop or laboratory, and that all ships having these gears have experienced uninterrupted, trouble-free operation of the reduction gears in service.

Additional Corroborative Cases

Since 1950 there have been a number of instances of difficulty with high-speed gears, and nearly all of the first reduction cases are explained by the theory.

Minor Scuffing on LSD (Landing Ship Dock) Main Reduction Gears. On the first two ships of this class, after the sea trials minor scuffing appeared at the gap ends of the helices of the high-speed high-pressure pinion, on the *starboard* reduction gear only—in which case the apex of the helices trail, so that the inside of the vee on the pinion is loaded. No difficulty or load concentration was experienced on the port gears. The scuffing was corrected by reshaving the pinions and the use of a mild EP oil (Navy symbol 2190-TEP).

Tooth Failures on Shop Tests of a Large Aircraft Carrier Gear. Of three high-speed pinions that developed fractured teeth on shop test, all three were loaded on the inside of the vee of the helices and the fractures were located at the gap ends of the helices. No failures occurred on the outboard ends of the helices nor for the case where the pinions were loaded on the outside of the apex.

Merchant-Ship Gears. There have been several unexplained minor difficulties on the first reductions of merchant-ship gears. All of these have been at the gap end of the helix, and in every instance the loading was on the inside of the apex of the helices. Although the merchant-ship gear transmits only about one half the horsepower per mesh as compared with a naval propulsion gear, the pitch-line speeds are as high, the pinions are as large, and the high-speed wheels are much larger than those on Navy gears. These factors all contribute to establishing the conditions for thermal instability.

Methods of Avoidance of Thermal Instability

A complete description of a disease as well as its symptoms and causes is interesting but of no great help to the patient. But, when the right diagnosis is made, the treatment and cure become evident. The theory of regenerative heating, developed herein, has led to the development of methods of design and manufacture that will eliminate the occurrence of thermal instability, and will produce gears of great reliability and low specific weight.

The most important factor is the direction of loading with respect to the vee of the helices. For conventional marine first-reduction gear trains, a solution can be obtained simply by selecting the hand of the helices so that the apex leads for ahead rotation. For single-screw ships, this can readily be done. However, for multiple-screw ships with opposite rotation of propellers, it is undesirable to have opposite-handed gear trains.

In this case, when the apex must trail, thermal instability can be controlled by center-gap cooling and the selection of tip and root relief.

Center-gap cooling by special oil sprays or other means such as improved conduction paths is very effective in limiting the build-up of stress at the gap due to barreling of the pinion or gear, and hence careful design of center-cooling schemes is essential for high-speed gears. When other unfavorable conditions are cascaded, center-gap cooling alone is not sufficient to prevent thermal instability, so that attention must be directed toward reducing rather than dissipating the heat generated at the mesh.

The heat generated at the mesh can be controlled and minimized by careful selection of tip and root relief on the tooth profiles. The reduction in tooth pressure near the extremities of the tooth (where the sliding velocities are highest) changes the shape of the heat-loss curve in Fig. 17 by blunting the peaks and thus the area under the curve, or total heat generated, is markedly reduced. The data in the section on unfavorable laboratory results showing the difference in performance between the A and B units make it clear that controlled tip and root relief are essential if successful operation is to be obtained when the direction of loading is adverse. The tip and root relief should increase gradually with distance from the pitch line and not abruptly so as in effect to stub the tooth. It is essential that tip and root relief be limited in amount to small values, for otherwise the tooth bending stress and contact stress will be increased excessively.

Acknowledgments

Mr. J. MacInnes, Senior Manufacturing Engineer, made valuable contributions during the course of this investigation in the gear-test department. The strain gage instrumentation was developed and operated by Mr. V. Donato. The Steam Division Development Laboratory erected and operated the experimental gear test facility.

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DISCUSSION

Milton S. Berg[2]

This paper is of particular interest in that it furnishes an explanation for some of the tooth breakage of high-speed gearing which has occurred in recent years. This tooth breakage on double helical gears, constructed in the postwar years, has been characterized by fracture of teeth at the gap end of either forward or after helices.

A review of such casualties for postwar construction naval ships indicates that two thirds of such casualties could be explained by the information furnished by this paper. However, breakage of teeth at the gap has been a rarity for the designs which were manufactured prior to and during war years. There would appear to be some correlation between these facts.

Except for a higher level of tooth loading the chief difference between present and previous designs is gear wheel web construction. Web construction is not referred to in the paper although Fig. 11 illustrates the effect of speed. This illustration indicates that at the gap there is a distinct relationship between speed and stress level. The older design gear wheels usually had four webs arranged as a double vee. This construction apparently minimized the differential expansion across the face caused by centrifugal force. Reference to four-web construction is not intended to imply that other construction is not satisfactory. There are numerous recent designs of two web gears which have been fully adequate, both during overload shop testing and in ship operation. The criteria for wheel webs should be a design which minimizes excessive localized rim growth which is apparently basic in regenerative heating.

It would seem that a combination of satisfactory gear wheel

[2] Bureau of Ships, Washington, D. C.

design, profile modification, and adequate oil sprays can avoid regenerative heating and consequent thermal instability and negate the importance of the direction of loading in respect to the vee of the helices.

C. H. McDowell[3,4]

Test experience at the Naval Boiler and Turbine Laboratory has failed to show the thermal instability phenomena described in this paper. However, similar conditions in less serious form could possibly remain undetected without the aid of the small but very potent strain gage, without which the authors could have been seriously pressed to explain the circumstances of failure.

A very interesting aspect of the authors' experience is the mode of gear tooth failure, which appears very similar in nature to many which have occurred at the Laboratory. This mode of fatigue failure, with origin on the tooth at the lowest portion of the active profile, is not covered by conventional tooth bending stress formulas, which anticipate fracture through the tooth fillet rather than the active portion of the profile. Failures with origin of fatigue on the active profile may be lightly discarded, attributing the cause to weakening of the material from fine pitting or crazing of the surface which is associated with pitting. However, this reasoning has not explained all of the Laboratory failures which, in instances, occurred with no evidence of surface distress to trigger the failure.

The mechanism for failures of this type may possibly be explained in the following manner. Maximum normal compressive stresses for the contact band between helical pinion and gear may be computed from the well-known Hertzian equation, assuming negligible sliding friction. It may also be shown that a small cube of material at the surface is stressed compressively in a plane perpendicular to the line of action but in a direction normal to the contact line with magnitude equal to the normal compressive stress. In the normal load cycle, this same cube of material, if near the lowest portion of the active profile, will later be highly stressed in tension as load progresses toward the tooth tip. Thus the cube of material on the tooth surface is alternately stressed, first in compression, under direct contact, and later in tension, under bending, when load transfers to the tooth tip. This alternating stress range is conducive to the start of fatigue failure. The probability of failure from this cause may be assessed with the aid of the well-known Goodman diagram.

Bending stress on the pinion tooth flank, in the area of contact with the gear tip, should be approximately two thirds of the maximum fillet bending stress or approximately equivalent to the indicated gage measurements in consideration of fillet concentration factors and reduction in bending moment at the higher point on the tooth flank. On this basis, a maximum tensile stress of approximately 37,000 psi would exist as deduced from averaged

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[4] The opinions presented in this discussion are private ones of the writer and are not to be construed as official or reflecting views of the Navy Department.

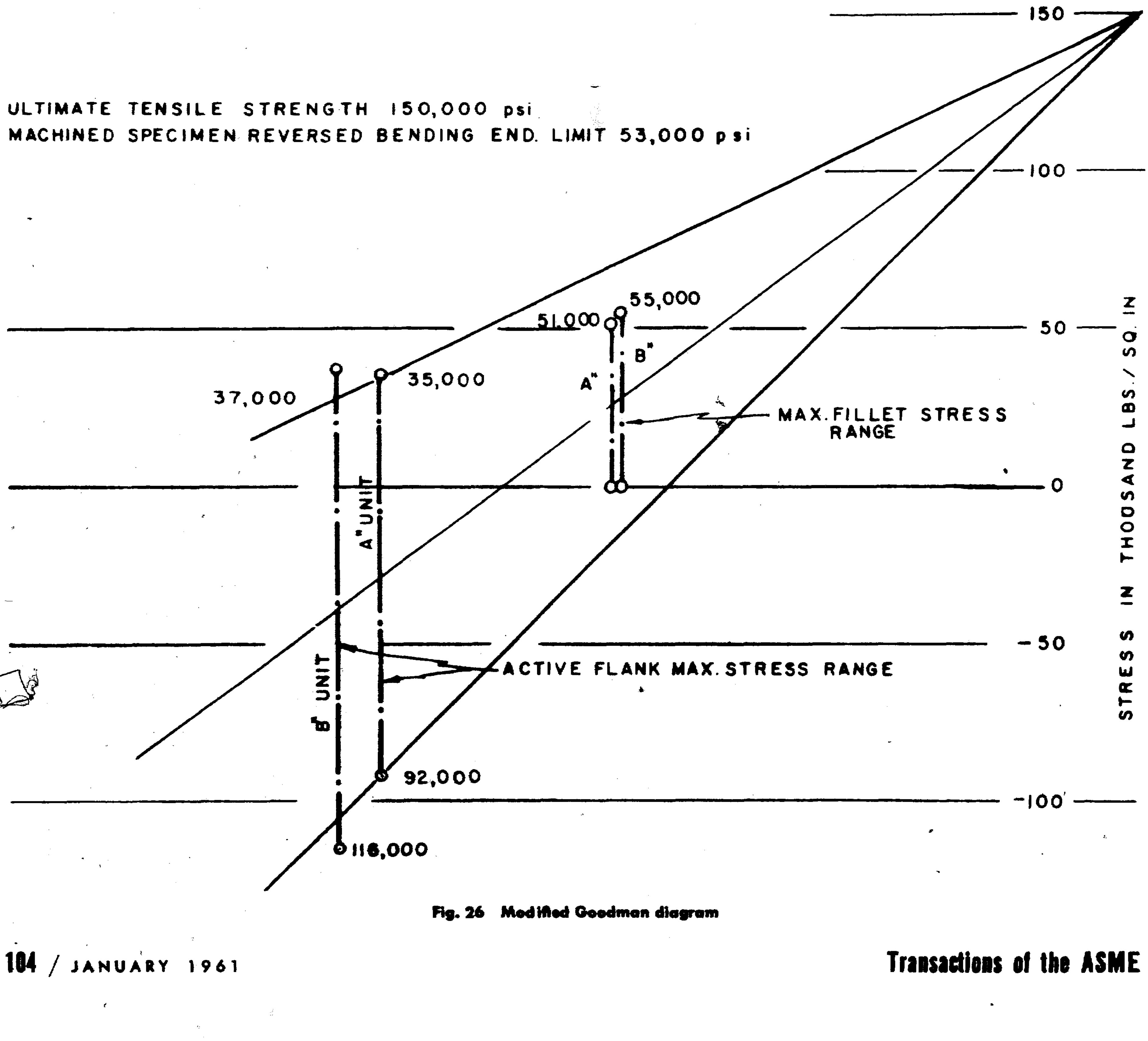

Fig. 26 Modified Goodman diagram

strain gage indication or 55,000 psi maximum in the fillet for the B unit. For the A unit, equivalent maximum values for the worst case are estimated to be 35,000 psi flank and 51,000 psi in the fillet. The compressive stress on the pinion tooth flank, computed for the area of contact with the gear tip and assuming full uniform contact, is approximately 65,000 psi at rated load. As influenced by a load concentration factor of 3, this value is raised to 116,000 psi for actual conditions in the B unit and 92,000 psi for a concentration factor of 2 in the A unit.

These stress ranges may be plotted on a modified Goodman diagram (Fig. 26) for the pinion material. The endurance limit range is determined by the diagonal lines drawn from the machined finish reversed bending endurance limits, of approximately 53,000 psi, to the ultimate tensile strength of 150,000 psi. It is seen that the indicated ranges plotted on the diagram exceed the boundary limitation and failure is to be expected on the tooth flank at the initial contact zone. For comparison, the maximum indicated fillet bending stress range has also been plotted, which is shown to fall well within the diagram limitations and fatigue failure through the tooth fillet would accordingly not be expected.

It is felt that the strain gage will, in the future, provide much additional valuable information. The authors are to be congratulated for their exceptional work in this field and for their material contribution to gear technology.

W. S. Richardson[5]

From a sense of honest conviction, rather than as a matter of politeness, I want to compliment the authors of this paper. Their development of a logical theory to explain observed phenomena and their practical, common sense evaluation of the theory and its application to high performance gearing make easy objective consideration of what may prove to be a very significant contribution.

Regarding the theory of thermal instability itself, the authors' arguments are persuasive and appear to be supported in a general way by the data presented, although the specific accuracy of much of this data, and particularly of the time-temperature curves on Fig. 16, may be open to question. From our own experience with similar propulsion gearing, we can add corroborating evidence of the same type of tooth contact patterns which lead them to the development of the thermal theory.

On a class of three naval combatant vessels having highly loaded gearing, in each case the starboard unit HP and LP first reduction elements with apex trailing exhibited, with startling consistency, the contact conditions predicted by the theory. At the gap ends of both helices the pinions showed extremely heavy bearing high on the teeth, some pitting, and a tendency to scuff at the tips. One fracture failure also occurred at this area. The meshing gears also had heavy bearing at the gap ends of both helices, but low on the tooth, and in some cases concentrated pitting well below the pitchline. Both the pinions and the gears often developed, in the same area, a matted brownish burned appearance, rather than the usual high polish.

Conversely, the first reduction elements of all three port units with leading apex, uniformly showed even bearing and good polish across the faces of both helices, with a slight tendency to a heavier bearing at the outer ends.

The combination of "barreling" plus axial expansion postulated by the authors would appear to explain these conditions, and there is little doubt that the effects would be cumulative when the apex trails and differential when it leads, as illustrated by Fig. 22 in the paper. However, there would appear to be no justification for considering the combination to be more than merely quantitative for any given set of conditions, or to interject a theory of regeneration or thermal instability. This would infer that beyond some unstated critical point, comparable to exceeding the critical mass of fissionable materials, the process becomes progressive and a failure inevitable. The authors do not state that in any run under adverse conditions, such as that represented by Fig. 24, which produced a failure in the "B" unit pinion in 6 hours, any progressive maldistribution of stress as a function of the time was actually observed.

It would seem rather more likely that under any given set of operating conditions, the gear set reaches a point where the heat flow, temperature distribution, and consequent dimensional distortions are stabilized, and, depending on their magnitude, either are or are not sufficient to produce failure in some specific number of contact cycles. Some support of this point of view is lent by the fact that very minor indication of the same contact variation described above has been noted in other propulsion gearing of the same type but loaded to something less than half the "K" factor.

The recent experiences described in the foregoing remarks would appear to corroborate the thesis advanced by Mr. Welch and Mr. Boron. Nevertheless, contrary experience of many years standing does provide a general atmosphere of skepticism that we feel should be maintained until considerably more confirming data are recorded, particularly since other explanations are within the bounds of possibility, and some pertinent questions concerning the thermal theory do come to mind immediately.

For example, the temperature data of Fig. 16 is unconvincing, and we would doubt the propriety of projecting the curves backward to zero time. Contrary to the author's evaluation they actually show little difference between the alternate positions of loading and, if anything, merely demonstrate the cooling effect of journals during shutdown. Bearing in mind the heat input from the journals at speed, and that heat dissipation is primarily accomplished by oil sprayed on the periphery rather than the end faces, there would be little reason for a temperature differential between center and ends to cause barreling. Further, if the temperature of the gears is substantially the same as that of the pinions, which the curves do seem to show, and which might be expected in spite of the additional number of contacts, since it is the pinions which are ordinarily sprayed with cooling oil, then axial expansion of pinion and gears would be the same and it would be immaterial whether the apex trailed or led. The effect of barreling only would be observed, but this was not what was demonstrated by the difference between port and starboard units.

These and other matters, such as the effect of gear proportions and alternate details of construction deserve further research before final conclusions are attempted. To illustrate the point, the gears mentioned earlier in these comments had approximately half the face of those listed in Table 1, and a single central web rather than two side webs, which should have changed the concentration characteristics materially, but did not.

However, the suggestions of the authors to keep the possibility of load concentration due to thermal distortion in mind and to give special thought to means for insuring uniformity of temperature within and between meshing elements are well taken.

N. A. Smith[6]

The authors are congratulated on their paper and the theory and data presented. No doubt this work will contribute significantly to continued progress in the development of light weight, high load gearing.

Shop tests on large CVA gears at General Electric have yielded mesh temperature variations which generally agree with data presented in the paper. Sufficient data were taken to indicate

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[6] Manager, Marine and Naval Gear Engineering, General Electric Company, Lynn, Mass.

that contact conditions do exist as expressed in the paper. It was considered to be sufficiently important to modify mesh nozzles in a manner to limit temperature differences both across pinion faces and between mating pinions and gears. It is suggested that the authors could obtain additional data from their test which would establish the best nozzling practice for cooling of all mesh conditions.

Nonuniformity of tooth contact resulting from expansion of the gear rim under centrifugal force has also been confirmed experimentally by General Electric. Wheels were rotated at various speeds while dimensional changes were recorded by instrumentation. It was found that the wheel structure can be designed to minimize nonuniform expansion.

The writer would like to remind the authors that, while their tests and theory point up an important element of tooth loading which must be accounted for, there is another extremely important factor which is helix angle mismatch. Helix mismatch may generally be considered a combination of helix angle relationship and straightness of helix angle.

Tooth failures experienced by the General Electric Company are confined to DL-1 and 2 propulsion gears on initial trials. Shop tests have yielded no failures.

The totally unexpected DL-1 and 2 tooth failures necessitated a detailed investigation of design and manufacturing history which included:

1 A review of materials. This resulted in greatly improved physical properties of high hardness material through special heat-treatments and forging practice. However, the final evaluation of the failures assigned minor responsibility to materials.

2 First reduction wheels were tested for distortion due to centrifugal force.

3 Theoretical study was made of the affect of nonuniform temperature distribution on contact characteristics. These studies were not confirmed until CVA gears produced actual temperature data.

4 Analyzed helix mismatch. This resulted in a solution of the failures. The DL units were the first fleet application to utilize 400 plus Brinnel through-hardened materials. They also were the first fleet units to use a deliberate helix angle mismatch to compensate for pinion torsion and bending deflections.

Failures occurred at the forward ends of aft helices on high-speed pinions and at the bases of helices on low-speed pinions. An analysis of these deflections will show that helix ends will be overloaded at the positions mentioned. It is interesting to note that the affects of both temperature variation and centrifugal force also tends to overload the forward end of the after helix.

An analysis of localized overload due to twisting and bending of the pinion points out the necessity of using a helix angle mismatch between pinion and gear. The mismatch is readily calculated for each specific design condition. A contact check at no load will show one end of each helix open. However, when the pinion twists and bends under torque load, the contact will spread uniformly across the faces.

The DL pinions were designed and manufactured with helix angle mismatch. It was discovered later that some helices had proud faces at the ends where the shaving cutter heeled out. These proud ends caused the failures. At the time of manufacturing and shop test the small variations of contact bearing were considered to be of minor importance. It was felt that flexibility of gear teeth would prevent serious overload. This was a mistake and particular point is taken here to disagree with the author's statement that flexibility of gear teeth can permit moderately large misalignment without danger.

Flexibility tends to create an allusion of apparently good contact as observed but which results in one end of a helix being overloaded.

(*a*) First, when helix angle alignment is poor between a pinion and gear, flexibility will allow bending of the tooth under load and a postload examination will in fact indicate good contact.

(*b*) Secondly, when the tooth bends nonuniformly along the face, it is possible to increase tooth root bending stresses radically, perhaps two to three times designed loading and at the same time the contact markings may appear acceptable. This overloading explains most of the helix end failures.

Successful gearing requires close control of helix angle matching. The amount of mismatch is a function of helix length to diameter, as well as torque. Therefore it is possible to experience large amounts of combined twist and bending on either high or low loaded gearing. In many cases where helix misalignment exists, the root stresses are not sufficiently high to cause breakage. If these gears are of low hardness material, there may be pitting at the loaded end of the helix.

Properly manufactured gearing requires:

1 Machine tools capable of producing helix angle corrections.

2 Inspection personnel and tools capable of measuring modified helix angles.

3 Observation of contact distribution through the load range to prove that full face contact does not exist until predetermined load exists.

F. A. Thoma[7]

The authors presentation of the theory of "regenerative thermal instability" is a noteworthy step in the progress of developing gearing for highly loaded high-speed transmissions. The full scale testing described in this paper provides valuable data to the gear designer in terms of a quantitive evaluation of the effects of misalignment, speed, center gap-cooling, and oil spray location on gear tooth bending stress. The designer should bear in mind when considering these data, that the results were derived from gears of a specific construction and geometry, and that a change in construction or geometry may well vary the results and in some cases even reverse them. The conclusion that it is desirable, wherever possible, to arrange the hands of the helices so that the apex of the pinion leads in the direction of loading, is undoubtedly true in a great many cases, however, it cannot be applied universally without considering the geometry and construction of the pinion and gear in question.

To illustrate this, consideration could be given to a high-speed gear of relatively narrow face width that is machined from a solid forging with a single central web supporting overhanging rims on both sides. It is probable that the cool central web would draw heat from the rim so that the ends of the rim are hotter than the center. Centrifugal force would cause the ends of the rim to expand more than the restrained center portion. This gear then would become "hourglass" shaped rather than "barreled" in operation.

Pinion geometry should be considered when deciding on a leading or trailing apex. For example, pinions having a high length to diameter ratio can be subjected to rather large torsional and lateral deflections. These deflections are additive on the helix nearest the coupling and concentrate load on the coupling end of the helix. They are subtractive on the helix away from the coupling. For a pinion such as this, operating temperatures may well be such that a trailing apex and its tendency to "hog the load at the center gap" will nicely offset the effect of torsional and lateral deflections to concentrate load at the critical coupling end of the helix

Another consideration should be the pinion journal temperatures. In a locked-train gear, such as described by the authors, the pinion meshes with two gears at 180-deg points. With this

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arrangement, the tangential and radial loads from one gear cancel those of the other. The resultant load on the pinion journals then is only the weight of the pinion which is very small. The journals in turn are sized only for the transmission of the required torque. This results in small, lightly loaded, cool running pinion bearings in locked-train high-speed drives. The cool journals will draw heat from the pinion body creating the barreled shape.

It would not be too great a stretch of the imagination to picture a pinion running at 12,000 to 14,000 rpm and engaging a single gear. In this case, the tangential and radial loads of the mesh must be supported by the pinion journals so they, in turn, will be proportionally larger than the journals of a locked train pinion. The resultant geometry could be such that the journals are nearly as large as the pinion pitch diameter. The high journal velocities and high bearing loads could produce temperatures equivalent to the mesh temperatures. In this case barreling would be absent leaving only the temperature effect of increased axial pitch.

Some of the foregoing required supposition, but it serves to illustrate that there are exceptions to most all rules.

The exceptions are further illustrated by the examination of three cases of tooth breakage in high-speed gears of our design and manufacture. The drives were all locked-train. The tooth breakage in two cases occurred on the pinion and in one case on a gear. Surprisingly, the breakage occurred at the gap in all three cases. One case fits the theory of regenerative thermal instability perfectly. The other two, however, had leading apexes which demonstrates that other factors can also cause tooth breakage.

In conclusion, I would like to repeat that the authors have made a valuable contribution in the continued development of high-speed gearing for which I extend to them congratulations and thanks.

Authors' Closure

Messrs. Berg, Thoma, Richardson, and Smith each stressed the importance of high-speed wheel design in controlling tooth load distribution, a consideration in which the authors heartily concur. Of particular interest in this respect is the description of high-speed wheels given in the next-to-last paragraph of Mr. Richardson's comments, this being the type of wheel mentioned by Mr. Thoma. It should be noted that, despite this marked difference from the design shown in Fig. 10, the behavior as described by Mr. Richardson closely parallels our experience during the shop and laboratory tests.

Mr. Berg cites the absence of failures of this general type in the gears designed prior to and during the war. These gears were of lower K-factor than the present naval gears, although the power transmitted was nearly as great. Thus for nearly the same horsepower, the size of the reduction gear was much larger, and the heat dissipated in the mesh bore a lower relation to the mass and volume of the gears and pinions. Because of the lower resistance to heat flow away from the mesh for the larger gear, the larger gear is much less likely to show a thermally regenerative condition than a smaller gear, if each is transmitting the same horsepower. (A large log is more difficult to set afire than is a small log--particularly if we use the same size match!)

Mr. McDowell, in his presentation of Fig. 26, has provided a sound and new explanation of why the tooth fracture crack begins on the active profile rather than in the tooth root, and we thank him for supplementing the paper with this explanation.

Mr. Richardson's comments on the time-temperature curves of Fig. 16 are well taken, for we are aware of the roughness of these data. In fact, when the paper was first prepared, this information was not included because of its very approximate nature. One of the reviewers of the first draft was most skeptical until he was later shown these curves, which convinced him of the plausibility of the theory. This experience led us to add the contact temperature data to the text.

The discussions by Mr. Smith, Mr. Richardson, and Mr. Berg add additional corroborative cases of tooth failures to the theory, at least in so far as the adverse gap-heavy loading of an apex trailing pinion is concerned. Mr. Smith is entirely correct in stating that effects of temperature variation, centrifugal force, and deflection due to twist are additive at the gap end of the aft helix of the high-speed pinion. However, it is believed by the authors that the deflection due to twist at this location is the least important of the above three components, and in this belief we appear to be at variance with Mr. Smith. The deflection due to twist can be easily calculated.

On highly loaded gears, we agree with Mr. Smith that good contact at full load can be misleading due to the flexibility of the teeth. Good manufacturing practice in this class of gearing requires successive test floor runs at $^1/_4$, $^1/_2$, $^3/_4$, and full torque to verify good tooth geometry and alignment.

The authors wish to thank the discussers for their general remarks on the paper and for the additional information they have contributed to the subject.