Tuning out torsional vibration

Pinpointing vibration sources prevents potentially serious machine failures.

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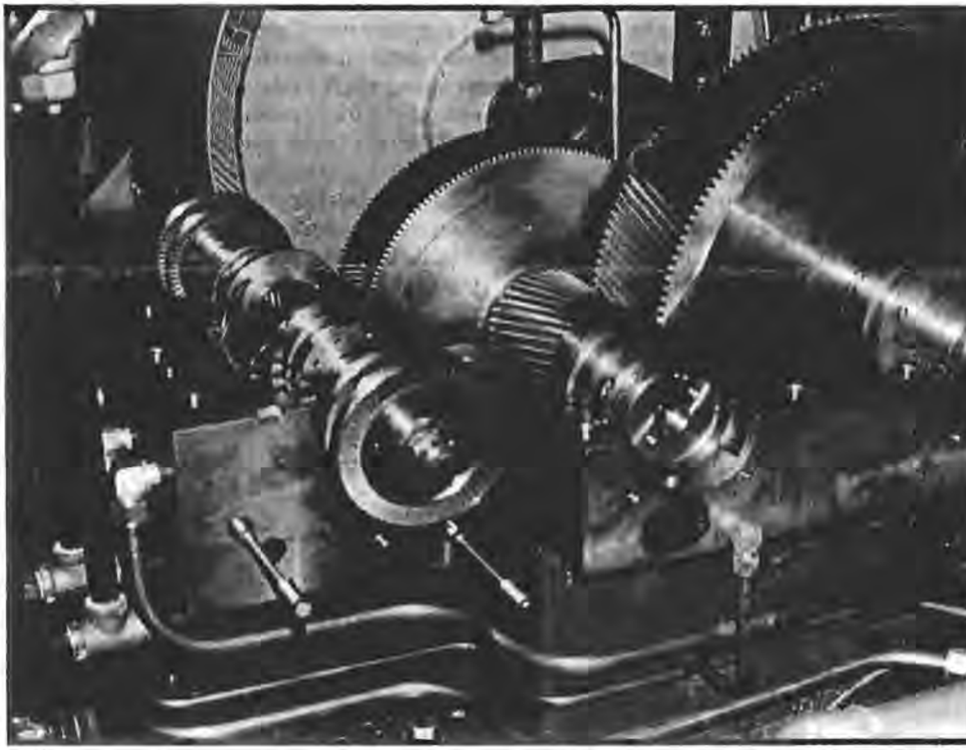


Gears, pumps, and motors are among the common sources of torsional vibration.

The first symptom of trouble in pumps, compressors, and other turbomachinery is often a broken shaft, gear tooth, or coupling, or excessive wear on gears and splines. The real culprit is torsional vibration which, by some accounts, is the leading cause of failure in turbomachinery drivetrains. Fortunately, proper design procedure almost always heads off trouble.

Problems usually occur because most engineering resources and software packages can determine a machine's natural frequencies, but few go the extra step of predicting torsional vibration problems. This article provides the design procedure necessary to rectify the situation.

UNDAMPED ANALYSIS

The first step in any torsional analysis determines a system's natural frequencies and mode shapes, usually with a lumped-parameter model where disks represent the system's inertial components and torsional springs represent shafts. Designers generally assume linear spring behavior with torque directly proportional to twist angle. Real systems also contain damping. However, damping complicates the model while barely affecting accuracy, so it is usually ignored.

The model should identify all significant inertias in the system, such as impellers, propellers, motor and generator rotors, gears, and coupling hubs. Then determine each element's mass polar moment of inertia. Manufacturers usually provide inertias for purchased components such as motors and couplings. Testing or analytical methods can generate the rest. For example, find the mass polar moment of inertia for a common inertia element, a hollow disk, from

$$J = \frac{\rho\pi}{32}(D_o^4 - D_i^4)L$$

Next, find torsional spring rates for the connecting shafts. The general equation for torsional stiffness is

$$k = \frac{GI_p}{L}$$

Torsional-vibration handbooks provide inertia and spring-rate equations for nearly every conceivable element configuration.

Converting actual hardware into idealized shaft and disk elements often requires judgment. In many cases, such as in splined connections, overlaps between shaft and disk elements blur the distinction between the two. These cases are normally handled by assuming that the shaft element deflects up to a location known as the point of rigidity within the overlapping region. Beyond this point, there is no shaft deflection. Torsional-vibration handbooks provide equations for locating this point for most common configurations.

For any system, the number of degrees of freedom equals the number of disks in the model. Any system that does not have a shaft element connected to ground has a trivial natural frequency of zero, representing rigid-body rotation. The number of nontrivial natural frequencies is, thus, one less than the number of disks in the model. Keep this in mind when selecting the number of disks.

Gear meshes are a special case. Because geared shafts run at different speeds, customary practice converts inertias and shaft stiffnesses to their equivalent values on the lowest speed shaft. The resulting single-shaft model has the same dynamic characteristics and natural frequencies as the actual system — much the same as combining resistors in series or parallel to obtain an equivalent electrical resistance.

An equivalent system assumes meshing gears are in continuous contact, so each mesh contributes only one degree of freedom to the system despite the fact that each mesh has two disks. For a given gear mesh, elements on the low-speed shaft do not change. However, transform elements on the high speed shaft with $J_e = JN^2$ and $k_e = kN^2$.

Use these equations, one shaft at a time, to replace two shafts by an equivalent one until all are referenced to the lowest speed shaft in the system. Element axial position on the equivalent and actual shafts is the same.

Hydraulic couplings also require special attention. They consist of two mechanically independent, radially vaned impellers. Torque transfers from the driving to the driven shaft by kinetic energy of the working fluid.

The two shafts carry identical torque, but slipping causes a slight speed reduction across the coupling. The equal torques mean there is no need to reflect inertias and spring rates across the coupling by the square of the speed ratio. For most applications, it's acceptable to treat hydraulic couplings as zero spring-rate elements that effectively divide an assembly into two independent torsional systems.

Analysis methods

After preparing the lumped-parameter model, determine natural frequencies using methods similar to those for

Torsional-vibration basics

A turbomachinery assembly can be modeled by a torsional spring connecting two inertias. One inertia represents the driving element, usually a motor or turbine, and the other corresponds to the driven load such as a compressor or pump. The torsional spring simplifies the interconnecting shafting.

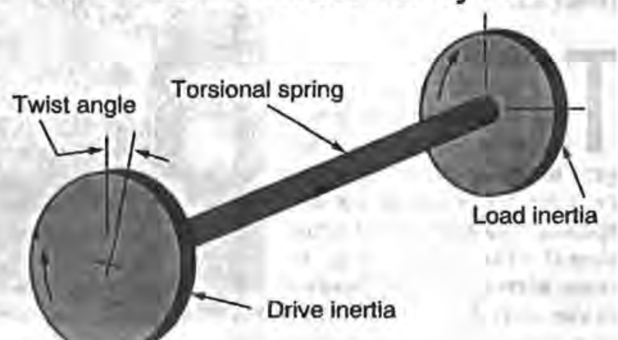
At steady speed, driving and load torques are equal and the two discs rotate at the same velocity. However, the shaft twists by an angle equal to the transmitted torque divided by its spring rate. This is referred to as the equilibrium position.

If drive and load torques are suddenly removed from the assembly, the twisted spring uncoils and twists an equal amount in the opposite direction. Oscillations would continue indefinitely without damping. The phenomenon, known as undamped free torsional vibration, is analogous to a linear mass-spring system. Regardless of initial conditions, the system always oscillates at a specific frequency — the undamped natural frequency. It is a system characteristic, a function of the inertias and shaft stiffness.

Forced vibration is illustrated by superimposing a sinusoidal torque on the steady torque of either disk. The resulting imbalance between driven and load torques causes all elements to vibrate about the equilibrium position. Consequently, all elements experience sinusoidal torque and speed fluctuations.

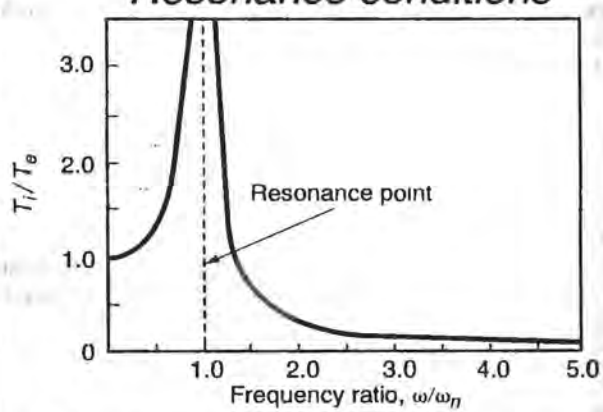
The system resonates when the driving frequency equals the system natural frequency. The ratio of induced shaft torque to excitation torque is theoretically infinite, producing large cyclic stresses in the shaft that often lead to shaft fatigue. Additionally, large generated peak torques can overload components such as gears, splines, and couplings. Therefore, the essence of torsional-vibration analysis identifies all resonance points and determines the system's ability to withstand them.

Two-inertia torsional system



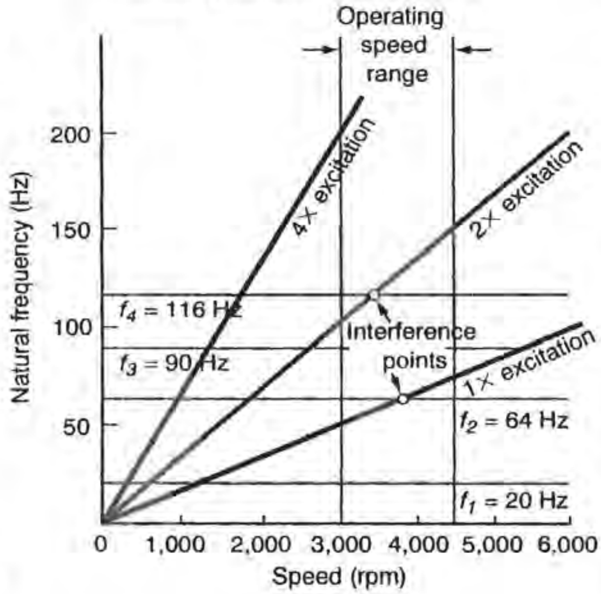
When running at steady speed, driving and load torques are the same. The twist angle equals the transmitted torque divided by the shaft spring rate.

Resonance conditions



Undamped response curve for a one degree-of-freedom system shows the possibility of large cyclic stresses in the shaft.

Representative Campbell diagram



This sample diagram represents a system that rotates between 3,000 and 4,500 rpm and has four natural frequencies. For any given interference point, the order number equals the natural frequency divided by critical speed, when both are expressed in the same units. The chart shows excitations having order numbers of one, two, and four. To generate an excitation line of a given order, arbitrarily select a speed on the *X* axis. Divide that number by 60 to convert to Hz, and multiply by the order number to determine the *Y* coordinate. Connecting this point with the origin by a straight line generates the excitation line.

any generic spring-mass system. For example, a single disk attached to a grounded spring has a natural frequency of

$$\omega_n = \left(\frac{k}{J_1}\right)^{0.5}$$

The form is identical to the natural frequency equation for a simple linear mass-spring system. Inertia and torsional stiffness are analogous to mass and linear spring rate.

For two disks connected by a shaft, natural frequency is

$$\omega_n = \left(\frac{k(J_1 + J_2)}{(J_1 J_2)}\right)^{0.5}$$

Closed-form solutions become unwieldy when a system has more than three degrees of freedom. Fortunately, a number of natural frequency programs are available today. However, computer solutions for natural frequencies occasionally generate errors, so a recommended practice checks results with independent hand calculations of the fundamental natural frequency. In the simplest case, the coupling's spring rate is an order of magnitude lower than any other shaft element. The coupling, therefore, takes almost all the deflection in the fundamental mode. This allows approximating the machine as a two-disk system by adding together all inertias on each side of the coupling, and calculating ω_n from the previous equation.

In general, when verifying computer output, reduce complex systems to a reasonable approximation that has three, or fewer, degrees of freedom. Because relatively small inertias have little effect on the fundamental frequency, ignore these disks and combine the shaft elements on either side as springs in series. Also, shafts with relatively large spring rates behave as if they were rigid in the fundamental mode. Therefore, discard these elements and add together the inertias on either side.

CAMPBELL DIAGRAMS

One of the primary objectives of torsional-vibration analysis is identifying all potential resonance points. This is no trivial task because most practical systems have numerous natural frequencies and multiple sources of excitation. A Campbell, or interference, diagram greatly aids the process.

Campbell diagrams plot natural frequencies as horizontal lines and the operating speed range as vertical lines. Upward sloping lines represent harmonics of speed that show potential excitations. Intersections between excitation lines and natural-frequency lines within the operating speed range are interference points that indicate potential resonances. Speeds corresponding to interference points are known as critical speeds.

A straightforward process determines speed range and natural frequencies. Generating excitation lines, on the other hand, requires considerable insight. Essentially, any mechanism that causes a periodic fluctuation in transmitted torque is a potential excitation source.

Excitations usually occur at integral multiples of the shaft speed. These multiples, known as order numbers, represent the number of vibrations during each shaft revolution.

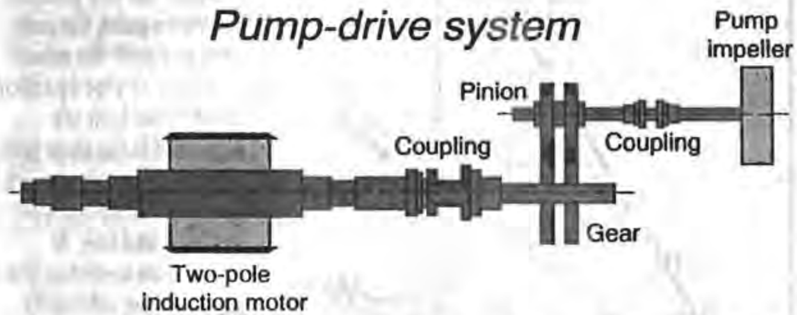
The most common excitations occur once and twice per revolution. Conditions such as rotating unbalance, eccentricity, and misalignment cause 1× excitation. Misalignment, ellipticity, and certain noncircular shaft cross sections such as keyways usually cause 2× excitation. Standard practice includes both these excitations in any torsional-system analysis.

Individual components such as the system drive, gear meshes, and load also generate excitations. Major sources are listed in the accompanying table and summarized below.

Gear excitations: Gears generate pulsations at several different frequencies. Gear-teeth imperfections and mounting errors can lead to unbalance, eccentricity, and misalignment and cause fluctuations with every shaft revolution; gear ellipticity generates torque variations at twice shaft speed.

Gears can also produce disturbances at their meshing frequency and higher harmonics of it. For each shaft, meshing frequency equals the number of teeth on the gear multiplied by shaft rpm. Therefore, the interference diagram for each shaft should include excitation lines corresponding to order numbers of one, two, and the number of gear teeth.

Impeller excitations: A vaned impeller that acts as either the load or driver is another common excitation source. This includes pump impellers, compressor and turbine rotors, and fans. When vanes pass a stationary object such as a volute or diffuser en-



A typical system where torsional vibration may be a concern involves a motor and pump operating on separate shafts, connected by reduction gears.

trance, pressure disturbances cause torque variations at blade-pass frequency. Casings can also generate excitations with an order number equal to the number of stationary vanes, as well as disturbances generated each time a rotor blade passes a stationary vane. For the latter case, order number n is

$$n = \frac{N_r N_v}{C_h}$$

Electrical machines: Because of the vast number of designs for motors and generators, manufacturers are the best source of information regarding excitations produced by electrical equipment. Most ac motors and generators running at constant speed produce fluctuations at 1× and 2× line frequency. In addition, many machines create oscillations with an order number equal to the number of magnetic poles. Dc machines generate relatively small excitations during steady operation, and can be neglected in many cases.

Variable-frequency drives: Variable-frequency drives permit variable-speed ac motor operation, and represent an additional source of torsional vibration. Varying electrical frequency controls motor speed. The resulting excitations are positively sloped lines with order numbers given by $n = n_p k_p N_p/2$. The number of pulses n_p is either 6 or 12 for all practical converters, and $k_p = 1,2,3...$

In contrast, some wound-rotor induction motors are controlled by what are referred to as static Kramer drives. These do not alter electrical frequency supplied to the motor, instead controlling speed by varying motor slip. These

Pinpointing torsional vibration

Generating Campbell diagrams for geared systems need not be confusing. To demonstrate the procedure, consider a system with a motor and pump operating on two different shafts. The motor is a two-pole induction unit driven by a standard variable-frequency six-pulse drive. Motor speed varies between 1,000 and 1,800 rpm. The gear mesh provides a 2.5:1 ratio, producing a pump speed range of 2,500 to 4,500 rpm. The pump impeller has eight blades while the pinion and gear have 10 and 25 teeth. The VFD is inactive below the operating range. Assume an undamped analysis shows there are four natural frequencies within the range of interest, at 50, 220, 340, and 500 Hz.

First, calculate order numbers for the motor and VFD. Because the motor has two poles, motor speed is

$$V = 60 f_e$$

Thus, if both are expressed in units of frequency, speed and electrical frequency are equal.

The order numbers for the motor shaft are generic 1× and 2×. Induction-motor pulsations occur at one and two times the electrical frequency, and are represented as 1× and 2× excitations on the Campbell diagram (duplicates of generic). With $n = 6$ and $N_p = 2$, the order numbers for the VFD are 6, 12, 18, and 24×. For clarity, only the first three will be used. Gear mesh excitation is 25×.

The corresponding order numbers for the pump shaft are generic 1× and 2×, impeller blade excitation is 8×, and pinion mesh excitation is 10×.

There are several options for plotting order numbers on Campbell diagrams. Perhaps the simplest is to prepare separate diagrams for each shaft. The natural frequencies are the same on both diagrams because the gear ratio has no impact on them. Excitation lines are simply drawn at the slopes indicated by the order numbers. Both diagrams are then checked for interferences.

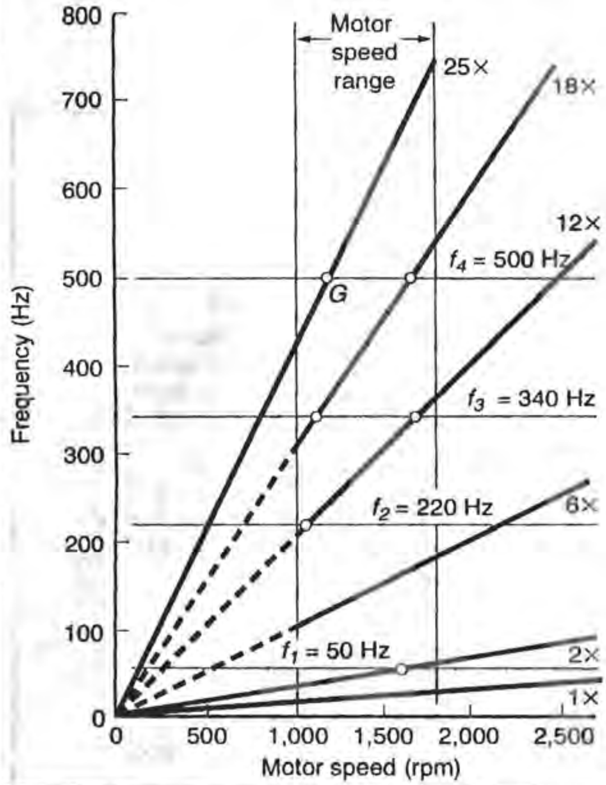
In this example there are six interferences in the motor shaft and four in the pump shaft. Any interference found on one shaft's diagram can cause vibration problems in the other shaft because we assume the gear mesh rigidly connects the two shafts.

Another method of plotting relevant information references data to motor speed. Motor excitations remain the same. However, to reference pump-shaft excitations to motor speed, multiply pump-shaft order numbers by the gear ratio. For instance, the general 1× excitation on the pump shaft is drawn with an order number of 2.5. Other pump shaft excitations are represented as 5, 20, and 25× lines.

The two methods reveal the same interference points. For instance, point *A* is an interference with the fundamental mode at a pump shaft speed of 3,000 rpm. Point *F* occurs at a motor speed of 1,200 rpm, which is equivalent to a pump speed of 3,000 rpm. Thus, the two points are identical.

Notice that the two individual diagrams have a total of 10 interference points while the combined diagram contains only nine. That's because points *G* and *I* represent interferences where the fourth natural frequency is excited by the gear meshing frequency. Therefore, these points are redundant and represented by single point *J* in the combined diagram.

Motor Campbell diagram



The figures above and below show interference diagrams for the motor and pump shafts. The third references data to motor speed and plots all interferences on a single diagram.

drives produce pulsations at harmonics of the slip frequency that appear as negatively sloped lines on a Campbell diagram.

Synchronous motor startup: Unlike induction motors, synchronous motors are not self-starting. Normally, squirrel-cage windings provide starting torque and damping during steady-state operation.

These windings accelerate the motor from standstill to slightly less than synchronous speed, when dc field voltage pulls the rotor into synchronism. This creates pulsating torques because synchronous-motor rotors contain salient poles that are magnetic protrusions enclosed by field coils. This asymmetry causes motor output torque to vary as a function of rotor position.

The stator's rotating magnetic field passing a rotor pole causes torque pulsations. Because the stator's magnetic field rotates at synchronous speed, excitation frequency is a function of the difference between synchronous speed and rotor speed, known as slip speed. Specifically, excitations occur at twice slip frequency where slip frequency is defined as

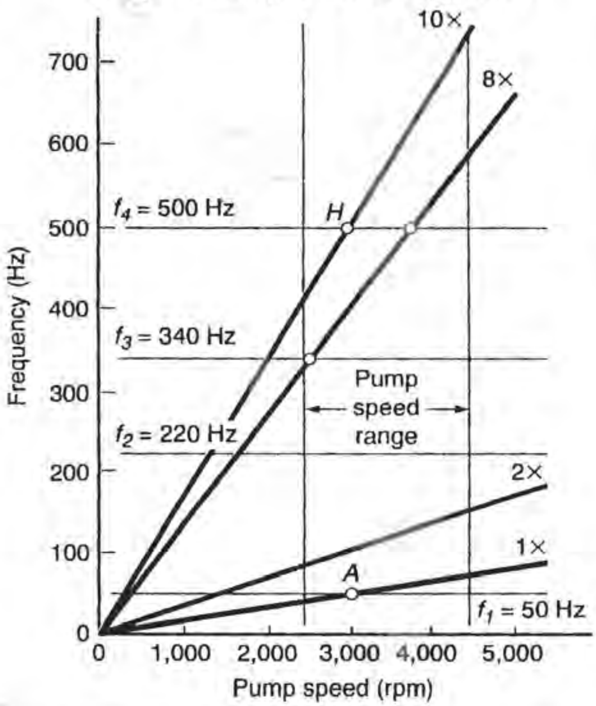
$$f_s = \frac{f_l(N_s - N)}{N_s}$$

and the motor's synchronous speed is

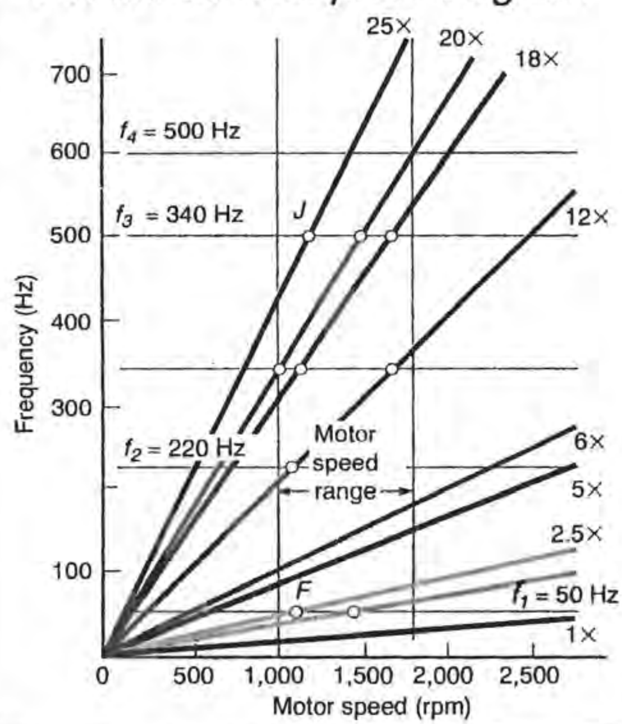
$$N_s = \frac{120 f_l}{N_p}$$

This shows that torque-pulsation frequency decreases as rotor speed increases. Thus, zero-speed excitation fre-

Pump Campbell diagram



Combined Campbell diagram



quency is twice the line frequency. As the rotor accelerates, excitation frequency decreases linearly until it reaches zero when the rotor is at synchronous speed.

After completing the Campbell diagram, identify intersections between natural frequency and excitation lines that represent true interferences. The user generally has two options for dealing with them. Either change the design to eliminate interferences, or further analyze the interference points to determine when they are worthy of attention. Experts typically recommend the second option because design changes are often costly or impractical. This topic is left for a future article.

The authors recently presented a 34-page paper on Torsional Vibration at the 25th Turbomachinery Symposium sponsored by the Turbomachinery Laboratory at Texas A&M University. The table on Excitation Sources, the Campbell diagrams, and the pump-drive example in this article are from that presentation. For a copy of the "Proceedings of the Twenty-fifth Turbomachinery Symposium," contact Turbomachinery Laboratory, Texas A&M University, College Station, TX 77843. Fax: (409) 845-1835. ■

Excitation sources and frequencies

Excitation source	Excitation frequencies
Generic 1× (unbalance, eccentricity, misalignment, etc.)	1× speed
Generic 2× (misalignment, ellipticity, etc.)	2× speed
Gear mesh consisting of pinion with n_p teeth mating with gear having n_g teeth.	**Pinion shaft:** 1× pinion speed 2× pinion speed n_p× pinion speed **Gear shaft:** 1× gear speed 2× gear speed n_g× gear speed
Impeller with N_r blades rotating inside casing with N_v stationary vanes	$N_r \times$ speed $N_v \times$ speed $n \times$ speed
Ac motor or generator with N_p poles. (Fixed frequency or static Kramer drive)	Line frequency 2× line frequency $N_p \times$ speed
Ac motor with N_p poles. (Variable-frequency drive controlling stator)	$0.5 \times N_p \times$ speed $N_p \times$ speed
Variable-frequency drive (stator frequency control) with N_D pulses driving ac motor with N_p poles	$0.5 \times N_D \times N_p \times$ speed $N_D \times N_p \times$ speed $1.5 \times N_D \times N_p \times$ speed $2.0 \times N_D \times N_p \times$ speed
Static Kramer drive with N_D pulses	$N_D \times$ slip frequency $2 \times N_D \times$ slip frequency
Synchronous motor (Fixed frequency drive)	$2 \times$ slip frequency

NOMENCLATURE

C_h = Highest common factor of N_r and N_s
D_i = Inside diameter, in.
D_o = Outside diameter, in.
f_e = Electrical frequency, Hz
f_l = Line frequency, Hz
f_s = Slip frequency, Hz
G = Shaft material shear modulus, psi
I_p = Area polar moment of inertia, in.^{4}
J = Mass polar moment of inertia, lb$_m$-in.^{2}
J_e = Equivalent inertia, lb$_m$-in.^{2}
J_f = Mass polar moment of inertia, lb$_f$-in.-sec^2
k = Torsional stiffness, lb$_f$-in./rad
k_e = Equivalent stiffness, lb$_f$-in./rad
L = Length or thickness, in.
N = Rotor speed, rpm
N_D = Number of drive pulses
N_G = Gear ratio (N>1.0)
N_p = Number of poles
N_r = Number of rotor blades
N_s = Synchronous speed, rpm
N_v = Number of stationary vanes
n = Excitation order number
n_g = Number of gear teeth
n_p = Number of pinion teeth
T_i = Induced torque
T_e = Excitation torque
V = Motor speed, rpm
ρ = Material density, lb$_m$/in.^{3}
ω_n = Natural frequency, rad/sec